

The Organization of Production and Economic Development

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Abstract

A formalization of the Coase-Williamson-Cheung theory of the firm is used to examine the trade-off between the firm and the market as institutions for organizing production in a dynamic, general equilibrium model with increasing returns to labor specialization. The model considers the interaction of internal and external transaction costs and the gains to labor specialization in determining important aspects of the organization of production including the degree of labor specialization, the size and specialization of firms and the pattern of interfirm trade. Endogenous growth is driven by capital accumulation and the evolution of the division of labor. The evolution of the organization of production in a growing economy is characterized by increases in the division of labor among individuals and firms, increases in firm and market size and increases in the complexity of the pattern of interfirm trade.

Section 1: Introduction

Recent work in the theory of endogenous growth has fostered a reexamination of Adam Smith's thesis that growth is driven by the evolution of the division of labor. Endogenizing labor specialization based on the trade-off between the gains to specialization and market transaction costs, models due to Yang and Borland (1991) and Becker and Murphy (1992) show that the gains to specialization may be used to generate aggregate increasing returns and endogenous growth in spite of the presence of diminishing marginal returns to capital. This contrasts with the main line of endogenous growth theory, which assumes that the properties of human or knowledge capital make the assumption of diminishing marginal returns to capital inappropriate.¹

The work on the division of labor has served to focus attention on the relationship between economic growth and the evolution of the organization of production. In particular, the growth process described by these models captures a number of changes in the organization of production associated with the more normative concept of economic development: increases in the specialization and interdependence of agents, increases in the number of goods, increases in market size and increases in the complexity of interpersonal trading relations.

However, in assuming that agents use markets to coordinate production through the price system and, thus, that market transaction costs provide an appropriate limit to the division of labor, these models ignore the significant role of firms in organizing

¹See, for example, Romer (1986) and (1990), Lucas (1988), and Becker, Murphy and Tamura, (1992). For examples of growth models based on the assumption that aggregate output is a linear function of the

production. The model developed here extends this work by developing a dynamic, general equilibrium model of growth due to the evolution of the division of labor which incorporates a formalization of the neoinstitutionalist theory of the firm developed by Coase (1937) and Williamson (1975). The model is used to consider the trade-off between firms and markets as institutions for coordinating production in a general equilibrium setting and to examine the evolution of the market, the firm and the division of labor in a growing economy.

The literature on the evolution of the division of labor is well suited as a framework for examining the neoinstitutionalist theory of the firm in a dynamic, general equilibrium setting. At the most general level, they share a primary concern with understanding the organization of economic activity. In addition, both proceed from a nano-economic perspective, taking the individual transaction to be the fundamental unit of economic analysis and the structure of transaction costs to be the primary determinant of the organization of economic activity. The literature on the division of labor applies a similarly disaggregate and heterogeneous perspective to production, basing its analysis on the individual productive task and the notion of task-specific capital.

According to Coase, the presence of market transaction costs implies that the price system fails to allocate resources costlessly, providing entrepreneurs an incentive to seek out an alternative means of organizing production. The institution of the firm arises because entrepreneurs opt to allocate some resources directly, allowing them to conserve on market transaction costs.

The resource allocation function of the entrepreneur is, however, both costly,

capital stock, see Jones and Modigliani (1990), King and Rebelo (1990), and Rebelo (1991).

generating what Coase terms internal transaction costs, and subject to diminishing returns, so that the cost of organizing an additional market transaction within the firm rises with the range of the firm's activities. Subsequent work on the internal organization of the firm due to Williamson (1975) supports the notion of diminishing returns by arguing that increasingly complex and costly hierarchical structures will be chosen as the firm expands the number of transactions organized internally. Rising internal transaction costs provide for a stable equilibrium regarding the range of a firm's activities, which is reached when marginal internal and external transaction costs are equal.

The Coase-Williamson theory of the firm provides the basis for endogenizing internal transaction costs as an increasing function of firm employment. In addition, as argued in the next section, external or market transaction costs are increasing in the number of market participants due to increases in transportation, information and contract enforcement costs.

Drawing on the work of Rosen (1983), the gains to specialization are modeled as arising due to the use of task-specific capital goods in intermediate good production, which introduces an "element of fixed costs of investment [making] it advantageous to specialize investment resources to a narrow band of skills and employ them as intensively as possible."² As a result, the gains to specialization are increasing in the capital-labor ratio, since a rise in an individual's stock of capital provides greater scope for exploiting increasing returns to intermediate good production.

In a static framework, the model illustrates the interactions between internal and external transaction costs and the gains to specialization in jointly determining the

²Rosen (1983), p. 44.

characteristics of the equilibrium organization of production, including the degree of labor specialization, firm employment and specialization, size and degree of specialization of firms, and the pattern of interfirm trade. In particular, the model examines the trade-off between firms and markets as institutions for coordinating production involving the division of labor and shows that both the institution of the firm and the phenomenon of exchange exist expressly due to the possibility of exploiting economies to the division of labor. As the coordination of production within the firm and through the market are costly activities, in the absence of increasing returns to specialization, there will be no firms and no interfirm or interpersonal exchange.

The dynamic model illustrates the role of the evolution of the organization of production in overcoming the effects of diminishing returns to capital and generating economic growth. An increase in the capital-labor ratio raises the gains to specialization resulting in increases in labor specialization, the size and specialization of firms and the number of market exchanges. Provided that marginal internal and external transaction costs rise relatively slowly with increases in firm and market size, the increase in economies to the division of labor will fully offset the effects of diminishing returns to capital, generating endogenous growth.

Rosen (1983), Barzel and Yu (1984) and Edwards and Starr (1987) examine the incentive for specialization. Baumgardner (1988), Kim (1989) and Yang (1990) consider the equilibrium degree of labor specialization in static, general equilibrium models, and Yang and Borland (1991) and Becker and Murphy (1992) generate models of endogenous growth due to the evolution of the division of labor. Young (1928) and Stigler (1951) analyze informal, dynamic models of firm specialization.

More recently, Yang and Borland (1995) have presented a formal, two-period, general equilibrium model of labor specialization and firm formation in the presence of exogenous internal and external transaction costs. Their use of exogenous transaction costs is based on Cheung's (1983) argument that by hiring labor to produce intermediate goods the firm substitutes a factor market, that for labor, for the intermediate goods market. Cheung's insight, however, does not imply that internal transaction costs are independent of the institutional nature of the firm. If, as argued by Coase (1937) and Williamson (1979), a labor market contract provides the firm not with a fixed quantity of services but with the authority to direct an individual's work effort, the efficiency with which a firm's bureaucratic structure carries out that task will have a direct impact on the cost of employing labor.

In addition, in Yang and Borland impose one-time, fixed cost to market and firm formation. This has two consequences. First, if their model were extended to consider three or more time periods, the division of labor would be static after the second time period.³ Second, firms become less specialized as the division of labor increases. That is, they predict vertical integration rather than vertical disintegration.⁴

The remainder of the paper is organized as follows. The next section lays out and solves the static model. The third section examines the effects of capital accumulation and develops a simple dynamic model illustrating the evolution of the market, the firm and the

³As Yang and Borland (1995) note, "Without the fixed transaction cost...the condition for a dynamic equilibrium with the gradual evolution of the division of labor will not be satisfied...." [Footnote 4, page 23]

⁴This difference of results also depends on the notion of specialization employed in each model. Yang and Borland (1995) model specialization as decreasing household self-sufficiency, that is it refers to the number of goods produced for self-consumption. In the model presented here specialization is an attribute of the labor market, referring to the range of tasks an individual performs in her job, and thus affects internal transaction costs.

division of labor over time. The last considers some ties to other areas of analysis and provides a brief conclusion.

Section 2: The Static Model

Section 2.1: Basic Functions

There are N *ex ante* identical individuals, each with an exogenously given stock of capital, h , and a continuum of productive tasks arranged along the unit interval and indexed by a , where $a \in [0, 1]$. There is a one-to-one relationship between tasks and intermediate goods, and the (measure of the) range of tasks undertaken by an individual is n . Labor specialization is inversely related to n : a worker is more specialized if she concentrates her productive efforts on an narrower range of tasks.

Each worker allocates her resources evenly among the tasks she undertakes and produces the same quantity of each intermediate good. Gross output per capita, defined as the sum of intermediate good outputs prior to the inclusion of internal and external transaction costs, is increasing in the degree of labor specialization and capital per head.

We have

$$(1) \quad y = y(n, h),$$

where $y(n, 0) = 0$, $y_n(n, h) < 0$, $y_h(n, h) > 0$, $y_{hh}(n, h) < 0$, $y_{nn}(n, h) < 0$. The derivatives of equation (1) imply that per capita gross output is increasing in labor specialization and the

capital-labor ratio and subject to diminishing marginal returns to capital, and that the marginal product of capital is increasing in labor specialization.⁵

Firms engage in two activities. First, each firm hires and coordinates the productive efforts of specialists to produce an intermediate composite good. One unit of the intermediate composite good is produced by combining one unit of each of the intermediate goods produced by each specialist within the firm. The length of the intermediate composite good, z , is defined to be the (measure of the) number of intermediate goods produced by a firm's employees and is inversely related to the degree of specialization of the firm.

In coordinating the production of specialists, the firm incurs internal transaction costs, which include management and monitoring costs, losses due to principal-agent conflicts and the like. In keeping with the earlier discussion of Coase (1937) and Williamson (1975), internal transaction costs are assumed to increase at a rising rate as a function of firm employees. In addition, self-management is assumed to be costless, so that internal transaction costs are zero when a firm has only a single worker. Therefore, we have

$$\begin{aligned}(2) \quad & I = I(L) \Rightarrow 0, \\ & I'(L) > 0, \\ & I''(L) > 0, \text{ and} \\ & I(1) = I'(1) = 0,\end{aligned}$$

⁵ In Appendix A these restrictions are shown to hold for per capita gross output when production is separable across tasks and intermediate goods are produced using Cobb-Douglas technology with diminishing marginal returns to task-specific capital.

where L is the number of a firm's employees.

Second, firms produce the final composite good, which may be consumed or, in the dynamic version of the model, invested. One unit of the final composite good is produced by combining one unit of each of the intermediate goods. A firm may produce the final composite good by producing the entire range of intermediate goods. Alternately, a firm may specialize, producing a subset of the intermediate goods, and rely on interfirm trade to obtain those intermediate goods it does not produce internally. This allows the firm to conserve on internal transaction costs. Intermediate composite goods are production substitutes for the intermediate goods which comprise them.

Positive external transaction costs imply that interfirm trading groups will consist of firms that produce non-overlapping intermediate composite goods. In addition, each firm will trade with every member of its group, as it must obtain the full range of intermediate goods in order to produce the composite final good. Integer problems regarding the number of firms in an interfirm trading group are ignored.

By engaging in interfirm trade, a firm incurs external transaction costs, which reflect the transportation, information and contracting costs incurred in conducting market transactions. External transaction costs are assumed to be a function of the number of a firm's trade partners. Assuming that firms are geographically dispersed, the average distance between trading partners will rise with the number of firms per market. An increase in the number of a firm's trade partners also increases the number of traded intermediate goods, which as Coase (1937) suggests will tend to increase information costs associated with discovering what the relevant prices are.

These considerations suggest that marginal external transaction costs are rising in the number of a firm's trade partners.⁶ In addition, external transaction costs are zero for a firm producing the entire range of intermediate goods. Thus, we have

$$(3) \quad E(J) > 0, E'(J) > 0, E''(J) > 0 \text{ for } J > 1, \text{ and} \\ E(1) = E'(1) = 0,$$

where E is the firm's total external transaction costs.

There are two reasons for believing that marginal external transaction costs might fall given a rise in the number of firms per trading group. First, we might expect the market power of specialized firms to decrease as the number of firms in each trading group rises, leading, in turn, to a decrease in bargaining costs associated with contract formation. This issue is not addressed directly in the model. Rather, following Yang and Borland (1991), the complications which the market power by specialized agents introduces into the analysis are avoided here by assuming that all contracts are negotiated prior to specialization decisions.⁷ Since agents are *ex ante* identical, no worker or firm has an advantage in the production of any particular set of intermediate goods at the time of contract negotiations. As a result, contracts reflect opportunity costs and composite intermediate goods (of equal length) are traded on a one-for-one basis.

The second concern regards the possible existence of significant economies of

⁶As indicated by equation (13), this is also a necessary condition for the model to generate an interior solution. If marginal external transaction costs are falling in J , the gains to specialization will outweigh the costs for all $n < 1$, implying $n = 1/N$.

⁷Becker and Murphy (1992) avoid the issue of market power by considering the social optimum. For an analysis of the impact of market power on specialization decisions see Baumgardner (1988).

scale to the transaction technology, such as might result from the public nature of infrastructure associated with transportation, information and contract enforcement. In fact, the formulation of external transaction costs given in equation (3) provides considerable scope for scale economies. In particular, external transaction costs are independent of the volume of trade. Thus, for example, a firm which doubles the quantity of intermediate goods traded while holding the number of market transactions constant will see a fifty percent reduction in external transaction costs per unit traded.

Furthermore, while information and contract enforcement costs may depend more on the number of trades than the volume of trade, provided the elasticity of marginal external transaction costs *per trade* with respect to the number of trades is less than unity, total marginal external transaction costs will be increasing in the number of trade partners as indicated in equation (3).

It is argued here that equations (1), (2) and (3') indicate a symmetric equilibrium across firms and individuals in L , z , and n . Within a given trading group marginal external transaction costs are uniform across trade partners, and as noted earlier, in equilibrium firms will equate marginal internal and external transaction costs. Thus, each firm in a given trading group will have the same number of employees, L , and degree of specialization, z .

The free mobility of firms across trading groups implies that L , z and n must be uniform across firms in different markets as well. Assume, for example, that firm size differs across trading groups. By equation (2) the larger firms will have higher marginal internal transaction costs and, since firms equate marginal internal and external transaction costs, these firms will also have higher marginal external transaction costs. To

compensate for these higher costs, the gains to specialization must be higher for workers in these firms as well. Otherwise, there will be an incentive for firms to move to the trading group with lower external transaction costs. This, however, cannot be the case, as the gains to specialization are determined by variables that are uniform across individuals.

Symmetry implies the following identities: the number of intermediate goods produced by a firm is given by $z = 1/J$; the number of intermediate goods produced within the firm is the number produced by each worker multiplied by the number of workers, $z = Ln$. Using the identity, $J = 1/z$, (3) can be rewritten as a function of z :

$$(3') \quad E(z) > 0,$$

$$E'(z) = -[dE/dJ]/z^2 < 0, \text{ and}$$

$$E''(z) = 2z^{-3}dE(J)/dJ + z^{-4}d^2E(J)/dJ^2 > 0, \text{ for } z < 1, \text{ and}$$

$$E(1) = E'(1) = 0.$$

Equations (1), (2) and (3') are denominated in units of the final composite good, which is taken as the numeraire. The firm's net output, Y , is equal to the sum across workers of gross output per capita less internal and external transaction costs. Using equations (1), (2) and (3') and the identity $z = Ln$, we can derive Y as a function of L , n and h :

$$(4) \quad Y(L, n, h) = y(n, h)L - I(L) - E(Ln).$$

Section 2.2: Optimization

The real wage, w , is determined in the labor market and, thus, taken as given by individual firms. As capital is assumed to be owned by labor, w interpreted as the return to an individual's labor and capital services taken together. Thus, firms choose L and n to maximize profits taking h and w as given. Each firm may employ a minimum of one and a maximum of N workers, so that $L \in [1, N]$. Similarly, since the continuum of intermediate goods is of unit length and since there are a total of N workers, n is constrained by $n \in [1/N, 1]$. Integer problems are ignored. The firm's objective function is, thus,

$$(5) \quad \pi(L, n, h, w) = y(n, h)L - I(L) - E(Ln) - wL,$$

where π is the firm's profit.

The firm's optimization problem is given by

$$(6) \quad \max_{L, n} \pi(L, n, h, w), \quad \text{s.t. } L \in [1, N], n \in [1/N, 1].$$

The Kuhn-Tucker conditions for equation (6) are

$$(7) \quad L \geq 1, \delta\pi/\delta L \leq 0 \text{ and } (L-1)\delta\pi/\delta L = 0$$

or

$$L \leq N, \delta\pi/\delta L \geq 0 \text{ and } (L-N) \delta\pi/\delta L = 0,$$

and

$$(8) \quad n \leq 1, \delta\pi/\delta n \geq 0 \text{ and } (n-1)\delta\pi/\delta n = 0$$

or

$$n \geq 1/N, \delta\pi/\delta n \leq 0 \text{ and } (n-1/N)\delta\pi/\delta n = 0.$$

The second order conditions for an interior (unconstrained) solution of (7) and (8) to be a local maximum are

$$(9) \quad \delta^2\pi/\delta L^2 = -[I''(L) + n^2E''(Ln)] < 0, \text{ and}$$

$$(10) \quad \delta^2\pi/\delta n^2 = y_{nn}(n, h) - LE''(Ln) < 0,$$

respectively. Here (9) is true from (2) and (3'), while (10) may or may not hold as no assumptions regarding the sign or magnitude of $y_{nn}(n, h)$ are imposed.

Section 2.4: Existence, Characteristics and Stability of Equilibria

Equations (7) and (8) provide a system of two equations in three variables, L , n and w . A third equation is generated by assuming that firms bid up the price of labor until it is equal to per capita net output, implying that a zero-profit condition will hold.

Assume, for the moment, that the second order condition in equation (10) is met and the constraints on n and L are non-binding. Equilibrium values of n , L and w are then determined by a system of three equations:

$$(11) \quad \pi = 0: \quad w = y(n, h) - [I(L) + E(Ln)]/L,$$

$$(12) \quad d\pi/dL = 0: \quad w = y(n, h) - I'(L) - nE'(Ln),$$

$$(13) \quad d\pi/dn = 0: \quad y_n(n, h) = E'(Ln).$$

Using (11) and (12) to eliminate w , we obtain

$$(14) \quad [I(L) + E(Ln)]/L = I'(L) + nE'(Ln),$$

which together with (13) can be used to determine the equilibrium values of n and L given h .

Let AA and BB, shown in Figure 1, denote respectively the schedules defined by (13) and (14) in the n - L plane. Equation (13) indicates that firms choose n by equating the marginal gains to specialization, $-y_n(n, h)$, with marginal market transaction costs. The AA curve, thus, gives the optimal degree of labor specialization as a function of firm size. As shown in Figure 1, the AA curve lies entirely below the curve defined by $z = Ln = 1$ and intersects the line $L = 1$ to the left of $n = 1$, since $Ln = 1$ implies positive gains and zero costs to specialization, $-y_n(n, h) > -E'(l) = 0$, implying that firms have an

Figure 1A: Stable Interior Equilibrium in (n, L)

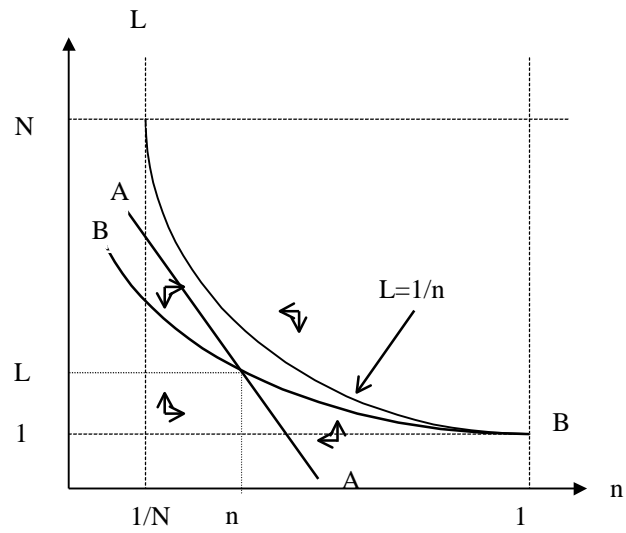
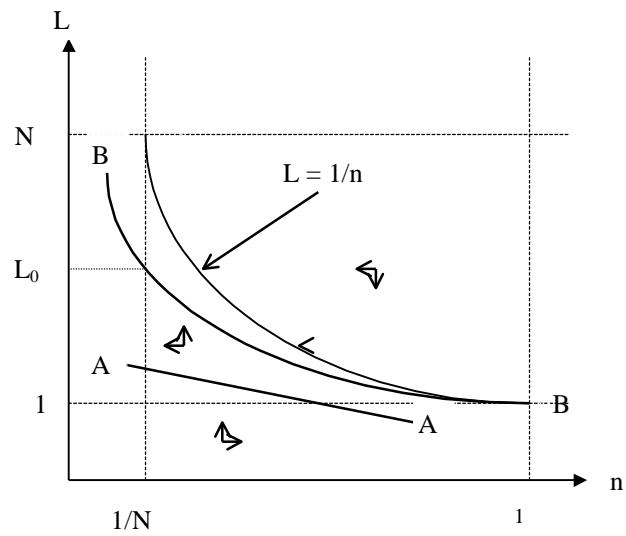


Figure 1B: AA lies below BB



Stable Equilibrium: $(1/N, L_0)$

incentive to increase labor specialization.

In addition, the AA curve is negatively sloped, as indicated by differentiating (13) with respect to L and n,

$$(15) \quad dL/dn|_{AA} = \frac{y_{nn}(n, h) - LE''(Ln)}{nE''(Ln)} < 0,$$

where the sign of (15) follows from (3') and (10). Intuitively, for a given value of n, an increase in L reduces the specialization of the firm, z, resulting in lower marginal market transaction costs. A decrease in n (increase in labor specialization) is then required to restore the balance between the gains to specialization and marginal external transaction costs.

Equation (14) is the familiar (long run) equilibrium condition that firms operate at the point at which the average and marginal costs of an input, in this case labor, are equal, that is at the minimum average cost. For a given degree of labor specialization, n, equation (14) indicates the optimal mix of markets and firms in organizing production is determined by the trade-off between internal and external transaction costs.

Differentiating (14) with respect to L and n, we find that the slope of the BB curve is given by

$$(16) \quad dL/dn|_{BB} = \frac{-LnE''(Ln)}{I''(L) + n^2 E''(Ln)}$$

$$= \frac{-L/n}{[1 + \xi(L, n)]^{-1}} < 0,$$

with $\xi(L, n) = \frac{I''(L)}{n^2 E''(Ln)} > 0,$

where signs follow from equations (2) and (3'). Thus, an increase in the specialization of labor increases the optimal level of employment. The reasoning behind this is that a decrease in n increases marginal external transaction costs. Therefore, a rise in employment is necessary to re-equate marginal internal and external transaction costs by increasing the former and reducing the latter.

As the denominator in equation (16) is always greater than one, the slope of the BB curve will lie between $-L/n$ and 0. In addition, equations (2) and (3') imply that the BB curve passes through the point (1, 1). Thus, the BB curve will lie between the line $L = 1$ and the curve defined by $Ln = 1$. As a result, the constraints on firm size, given by $L \in [1, N]$, are non-binding.

As illustrated in Figure 1A, the model produces an interior equilibrium provided the AA and BB schedules intersect for some $n > 1/N$. Interior equilibria are characterized by a positive degree of labor specialization and the use of both firms and markets to coordinate production. The first characteristic results from the fact that at $n = 1$ there are positive gains and zero costs to labor specialization. The second characteristic follows from the fact that the BB curve lies between the line $L = 1$ and the curve defined by $Ln = 1$, indicating that $L > 1$ and $z = Ln < 1$.

When (10) fails to hold or the AA and BB curves do not intersect for $n \in (1/N, 1)$, the marginal gains to specialization will be greater than the marginal costs for every point on the BB schedule. In this case, shown in Figure 1B, optimizing firms will choose the

smallest possible n , resulting in a stable equilibrium at the intersection of BB curve and the line $n = 1/N$. That is, there will be complete specialization of labor, with the BB curve determining the optimal mix of firms and markets in coordinating production.

As indicated in Figure 1A, the necessary condition for an interior equilibrium to be stable is that the AA curve is steeper than the BB curve at the point of intersection. In considering this condition, it is useful to derive a function for per capita total transaction costs, defined to be the sum of internal and external transaction costs per capita given optimal decisions regarding firm size. As the BB curve is monotonically decreasing, we can write optimal firm size as a function of n : $L = L(n)$, such that $(L(n), n)$ satisfies (14). Using this relationship, we can write per capita total transaction costs as a function of n :

$$(17) \quad t(n) = [I(L(n)) + E(L(n)n)]/L(n),$$

$$\text{with } t_n(n^*) = -E'[L(n^*)n^*] < 0, \text{ and}$$

$$t_{nn}(n^*) = \frac{LE''(Ln)I''(L)}{I''(L) + n^2E''(Ln)} > 0,$$

where n^* is the equilibrium value of n .⁸

From equations (15), (16) and (17) it follows that an equilibrium is stable provided

$$(18) \quad y_{nn}(n, h) < t_{nn}(n^*).$$

That is, an internal equilibrium will be stable provided an increase in the degree of

⁸See Appendix B1 for derivations.

specialization results in a greater increase in the marginal costs to specialization than in the marginal gains.

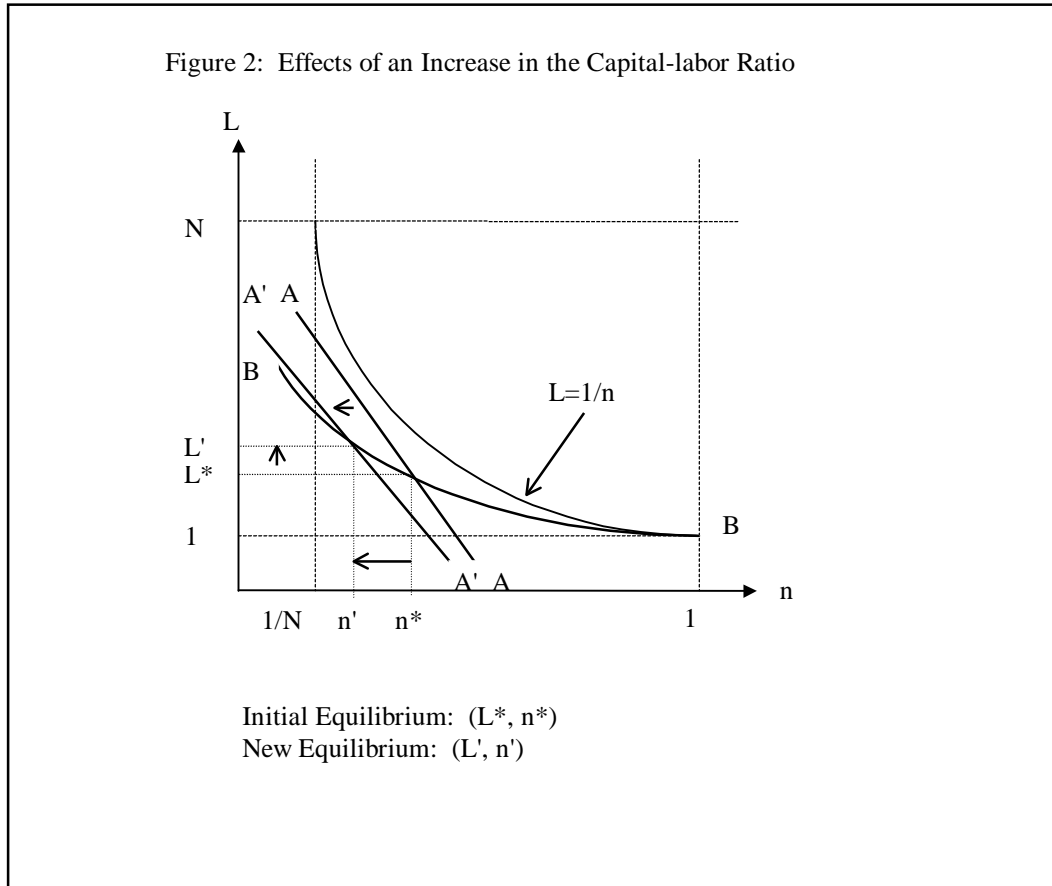
It is an interesting result of the assumptions made here that the autarkic equilibrium at $(n, L) = (1, 1)$ is not possible. In autarky the gains to specialization are positive while the marginal costs of exchange and firm formation are zero, $I'(1) = E'(1) = 0$. A corollary to this result is that the autarkic equilibrium obtains in the absence of gains to specialization. Given $y_n(n, h) = 0$, the AA curve collapses to a single point at $(L, n) = (1, 1)$. As this point satisfies equations (14) and (18), it constitutes a stable equilibrium. In autarky each "firm" consists of a single individual who produces the full-range of intermediate goods and is, thus, entirely self-sufficient.

This seemingly benign result has striking implications. In the absence of gains to specialization there is no incentive for agents to coordinate production. Thus, there are no firms, no markets and no interpersonal or interfirm exchange. By inference, the model implies that firms, markets and, thus, the majority of what we think of as economic activity exist due to efforts to extract the benefits of economies to specialization.

Section 3: Capital Accumulation, Specialization and Growth

This section analyzes the impact of capital accumulation on the primary variables of the model. We begin by considering the effects of an exogenous increase in the capital-labor ratio. Next, an informal dynamic model is constructed using a reduced form accumulation equation. The dynamic model is used to investigate the necessary conditions for endogenous growth and the existence of steady-state equilibria. Finally, we examine

the model's implications for the evolution market structure, the institution of the firm and the division of labor in a growing economy.



Section 3.1: Statics

As illustrated in Figure 2, an exogenous increase in the capital stock shifts the AA curve to the left, increasing equilibrium labor specialization and firm size. The (incremental) changes in the equilibrium values of n , L and z are given by

$$(22) \quad \frac{dn^*/dh}{dh} = \frac{-y_{nh}(n^*, h)}{t_{nn}(n^*, h) - y_{nn}(n^*, h)} < 0,$$

$$\frac{dL^*/dh}{dh} = \frac{-L/n}{[1 + \xi(L, n)]^{-1}} \frac{dn^*/dh}{dh} > 0,$$

and

$$\frac{dz^*/dh}{dh} = \frac{z^*}{n^*} \frac{\xi(L, n)}{1 + \xi(L, n)} \frac{dn^*/dh}{dh} < 0,$$

where the signs of the derivatives follow from $\xi(L, n) > 0$ and equation (19).⁹

The first equation in (22) indicates that the change in equilibrium labor specialization due to a given change in the capital-labor ratio will be large when the gains to specialization are sensitive to changes in the capital-labor ratio and the difference in the rates of increase of marginal per capita total transaction costs and gains to specialization is small.

According to the second and third equations, the degree to which a change in the equilibrium labor specialization is translated into increased firm size and firm specialization depends positively and negatively, respectively, on the rate of increase of marginal external relative to internal transaction costs. For example, if marginal internal transaction costs increase sharply relative to marginal external transaction costs, an increase in labor specialization will be reflected primarily in increased firm specialization rather than increased firm size. Put differently, market exchanges rather than the institution of the firm will be used to coordinate the more advanced division of labor. In addition, the third equation implies that the change in firm specialization is proportionately less than that of labor specialization and in the same direction.

⁹See Appendix B2 for derivations.

Thus, an increase in the capital-labor ratio increases the degree of specialization of both labor and firms, firm size, and the number of workers and firms in each interfirm trade group. In addition, as L rises and z falls marginal per capita internal and external transaction costs rise. These outcomes are driven by the fact that the gains to specialization are increasing in the capital-labor ratio.

Section 3.2: Endogenous Growth and the Organization of Production

By allowing the final composite good to be either invested or consumed and positing agents with an infinite time horizon, the static model developed in Section 2 may be extended to consider issues of economic growth. Of primary interest here is the ability of the model to generate endogenous growth in spite of the presence of diminishing marginal returns to capital in intermediate good production. This contrasts with the main line of endogenous growth theory which has argued that the special properties of either knowledge or human capital make the assumption of diminishing marginal returns to capital inappropriate.

An informal dynamic model is constructed by assuming that investment, equal to foregone consumption of the final composite good, conforms to the following function:

$$(27) \quad dh/y = f[w'(h)-\theta],$$

where $f(\cdot) > 0$ and $\theta > 0$ is the discount rate.

Equation (27) captures two properties of investment functions resulting from

dynamic utility maximization exercises with an isoelastic utility function. First, investment is a positive function of the difference between the marginal product of capital and the discount rate, which is the familiar Keynes-Ramsey rule. The second property, the gradual adjustment of the capital-labor ratio to its equilibrium value, implies consumption smoothing over time.¹⁰ It follows from equation (27) that the model generates endogenous growth if the marginal product of capital is greater than the discount rate and increasing in the capital-labor ratio.

The growth process is driven by mutually reinforcing increases in the capital-labor ratio and the division of labor. A rise in the capital-labor ratio has both direct and indirect effects on the marginal product of capital. The direct effect, due to the presence of diminishing marginal returns to capital, is negative. The indirect effect results from the impact of capital accumulation on labor specialization and that of labor specialization on the marginal product of capital.

Recall that the gains to specialization derive from the fact that specialization allows agents to concentrate their capital endowment and working time on a narrower range of tasks increasing both capital per task and the utilization rate of task-specific capital. As a result, the return to capital is increasing in the degree of specialization. In addition, as shown in the first part of this section, the equilibrium degree of specialization is rising in the capital-labor ratio.¹¹ Thus, an increase in the capital-labor ratio increases the gains to specialization, resulting in a higher equilibrium degree of specialization and an increase in

¹⁰For a similar reduced-form investment equation, see Becker, Murphy and Tamura (1992). See also note 14 below.

¹¹Mathematically, the increase in the return to specialization due to an increase in the capital labor ratio and the increase in the marginal product of capital due to an increase in labor specialization are identical, given by $-y_{nh}(n, h)$ and $-y_{hn}(n, h)$, respectively.

the marginal product of capital.

If the indirect effect is sufficiently strong to offset diminishing marginal returns to capital, the marginal product of capital will be increasing in the capital-labor ratio. A precise statement of this condition, derived in Appendix B, is given by

$$(28) \quad -y_{hn}(n^*, h) \left[\frac{-y_{nh}(n^*, h)}{t_{nn}(n^*) - y_{nn}(n^*, h)} \right] > -y_{hh}(n^*, h) .^{12}$$

The right- and left-hand sides of equation (28) show respectively the strength of the direct and indirect effects of capital accumulation on the marginal product of capital. The right-hand side is a measure of the strength of diminishing marginal returns to capital. The left-hand side is the product of the marginal increase in the return to capital due to an increase in the degree of labor specialization and the increase in the equilibrium degree of labor specialization due to an increase in the capital-labor ratio, as derived in equation (22). Thus, the model exhibits aggregate increasing returns provided the interaction between capital accumulation and economies of specialization is strong relative to the effects of diminishing marginal returns to capital.

Assuming the condition in equation (28) is satisfied, the size of the population provides a limit to the division of labor and, thus, to the growth process driven by capital accumulation and increasing returns to specialization. Let h_1 be the value of h for which $n^*(h_1) = 1/N$. At $h = h_1$ there is a single market in which labor is fully subdivided and each firm employs $L^*(h_1)$ workers, as defined by the intersection of the BB curve and the line n

¹²See Appendix B3 for derivation.

$= 1/N$. Beyond h_1 the accumulation of capital will continue to increase gross output, but will not increase the division of labor as labor is already fully subdivided. As a result, at high levels of the capital-labor ratio the model will be governed by diminishing marginal returns to capital. For $h > h_1$, we have

$$(29) \quad w'(h) = y_h(1/N, h) > 0, \text{ and}$$

$$w''(h) = y_{hh}(1/N, h) < 0, \text{ for } h > h_1.$$

Thus, the marginal product of capital is increasing in h for $h < h_1$ and decreasing thereafter. Note, in addition, that $w'(h)$ is continuous at $h = h_1$.¹³

Stationary equilibria occur at values of h for which $w'(h) = \theta$, which implies $dh = 0$.

Defining $\theta^* = w'(h_1)$, there are up to three stationary equilibria as follows:

$$(31) \quad \begin{array}{ll} \theta > \theta^*: & \text{one stable equilibrium at } h = 0, \\ \theta = \theta^*: & \text{one stable equilibrium at } h = 0, \\ & \text{one unstable equilibrium at } h = h_1, \\ \theta < \theta^*: & \text{two stable equilibria at } h = 0 \text{ and } h = h_2 \\ & \text{one unstable equilibrium at } h_0 \end{array}$$

where h_0 and h_2 are implicitly defined by $\theta = w'(h_0)$ for $h_0 < h_1$ and $\theta = w'(h_2)$ for $h_2 > h_1$.¹⁴

¹³From equations (29) and (B4), we have

$$\lim_{h \rightarrow h_1^+} w'(h) - \lim_{h \rightarrow h_1^-} w'(h) = [y_n(n^*(h_1), h_1) - t_n(n(h_1)^*)] dn^*(h_1)/dh = 0,$$

where the second equality follows from equations (13) and (17).

¹⁴Given a finite taste for consumption smoothing over time, the value of the threshold level of capital will be lower than h_0 . In particular, for values of the capital-labor ratio less than but sufficiently close to h_0 the discounted future gains along the path to the high-level equilibrium will outweigh the decrease in utility from investing when the marginal product of capital is less than the discount rate. As the difference in results obtained is quantitative rather than qualitative and because the analysis is simplified significantly, the reduced-form investment equation (27) is used here.

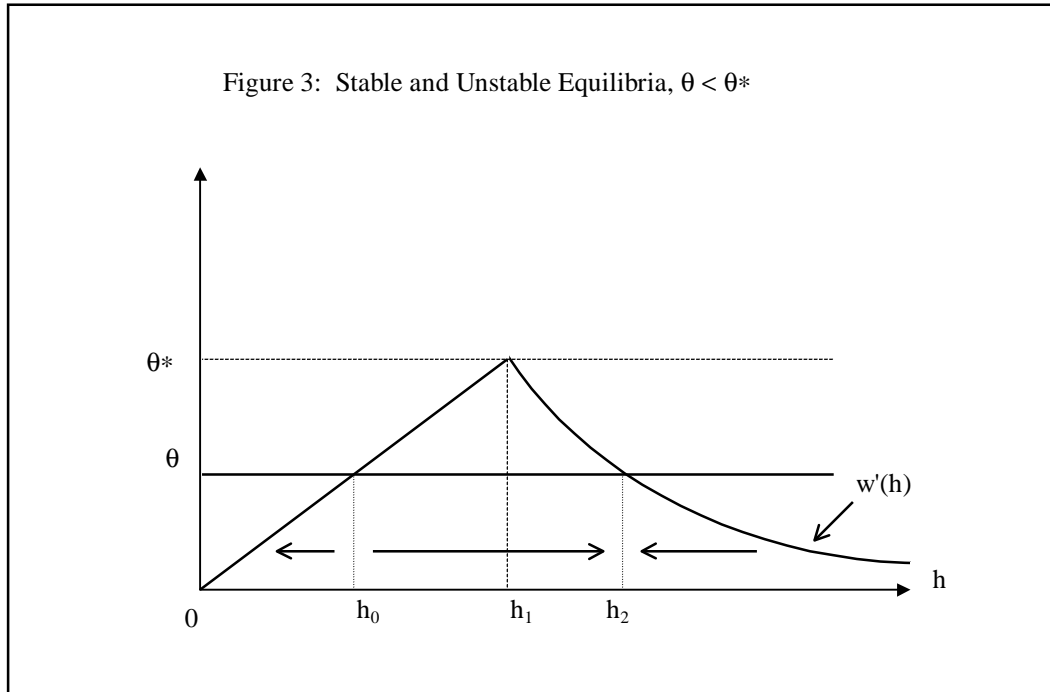


Figure 3 shows the stable and unstable equilibria for $\theta < \theta^*$. For non-equilibrium values of h , arrows indicated whether the capital-labor ratio is increasing or decreasing in time and, thus, whether an economy is converging to the high- or low-level equilibrium.

The implications for the evolution of market structure, the institution of the firm and the division of labor are found by combining the dynamics of accumulation described by Figure 3 and the results from the comparative statics of capital accumulation. An economy with an initial capital-labor ratio $h \in (h_0, h_1)$ will experience a gradual increase in the capital-labor ratio. The accumulation of capital raises the gains to labor specialization, implying increases in firm size, the number of a firm's trading partners, per capita marginal internal and external transaction costs and the specialization of firms and individuals. An economy with an initial capital-labor ratio below h_0 experiences these processes in reverse

as it approaches the low-level, autarkic equilibrium. Beyond h_1 , labor is fully specialized and the organization of production is static as the economy approaches the high-level equilibrium at h_2 .

Section 4 Comments and Conclusion

The primary purpose of this paper has been to investigate the relationship between the organization of production and economic development. In particular, it develops a model to investigate how the relationships between the gains to specialization and internal and external transaction costs jointly determine the equilibrium division of labor and use of firms and markets to organize production. In addition, in a dynamic framework, growth is driven by the interaction of capital accumulation and labor specialization. Endogenous increases in the gains to specialization result in the evolution of the division of labor, market structure and the institution of the firm, with the evolution of the institution of the firm being characterized by increases in firm employment and specialization.

The model portrays important facets of economic organization over a broad range of development levels, capturing the autarkic existence of agents in a traditional economy, the industrialization and commercialization processes experienced by growing, middle income countries and the highly advanced coordination of production and integration of markets seen in advanced, industrial economies.

The paper also touches on some issues that are important to other areas of analysis. There are natural ties to Krugman's (1994) work on complex systems and economic geography. An important characteristic of complex systems is the capacity for

self-organization, defined as the spontaneous emergence of higher-order structures among individual actors.¹⁵ Here, self-organization consists of the formation of firms and interfirm trading groups, with the complexity of organization within each increasing as the economy develops. This characteristic of economic development has its analog in the evolution of other complex systems, notably ecological and biological systems.

Second, by illustrating how the organization of production among firms changes as an economy develops, the model supports Allyn Young's (1928) assertion and recent evidence from Maddison (1994) that the firm should not be used as a unit of analysis for assessing economies of scale that result from gains to the division of labor. When marginal external transaction costs increase slowly relative to marginal internal transaction costs, an increase in the division of labor will be reflected primarily in increases in firm specialization and interfirm trade rather than in increased firm size. Similarly, the existence of large firms may reflect high external transaction costs rather than scale economies, as would seem to have been the case in Eastern Europe and the former Soviet Union.

This observation also bears on Knight's argument against the existence of Marshallian external economies arising due to productivity increases in supplier firms. Knight argued that these gains must then be internal to suppliers, and not a true source of productivity gains external to the firm. If, however, economic growth results in the reorganization of production among firms, then the gains to the specialization of labor will be realized partially through changes in the specialization of firms and the pattern of trade between contracting firms. Thus, the so-called external economies may arise precisely in

¹⁵In Krugman (1991), (1993) and (1994), this is illustrated by the spontaneous emergence of

the interstices between individual firms.

Finally, two comments regarding assumptions employed in the model are in order. First, by assuming that all contracts are negotiated prior to specialization decisions, the model neglects a significant source of external transaction costs. According to the analysis presented in Williamson (1979), when bounded rationality and a complex or uncertain environment prevent full contingent contracting, the existence of specialized investments can result in costly, *ex post*, small numbers bargaining between contracting firms. As the avoidance of these costs provides an incentive for industrial integration, the model as it stands may be thought of as overstating the degree of specialization of the firm.

Second, in modeling both internal and external transaction costs it is assumed that these costs are simply subtracted from gross output. In considering the losses that arise due to principal-agent conflicts, this may be an accurate depiction. More generally, however, transaction costs present an economic opportunity for those who can successfully reduce them. It follows that, to some extent, expenditures on firm management bureaucracies, certain government services and the transportation and information sectors provide an indirect measure of the resources devoted to abridging transaction costs. Thus, the rise of per capita internal and external transaction costs in a growing economy may be interpreted as increases in the service sector's share of national product.

manufacturing centers or cities among competing locations.

Appendix A

This appendix derives a parametric function for per capita gross output from a Cobb-Douglas production function for intermediate goods which uses task-specific inputs. The parametric function for gross output is shown to support the assumptions made in Section 2 regarding the signs of the derivatives of equation (1).

Let the set of tasks undertaken by an individual be given by $S \subseteq [0, 1]$. The (measure of the) number of tasks undertaken is given by n :

$$n = \int_0^1 b_a da,$$

where $b_a = 1$ if $a \in S$ and 0 otherwise.

There is a one-to-one relationship between tasks and intermediate goods: performing a task produces a quantity, y_a , of the intermediate good of the same index number. Intermediate goods are produced according to a Cobb-Douglas production function with arguments t_a and h_a , which are respectively the time and capital allocated to task a :

$$(A1) \quad y_a = A t_a^\alpha h_a^\beta,$$

where $\alpha, \beta \in (0, 1)$ and are uniform across tasks.

Each individual has a single unit of time and h units of capital which is allocated evenly among tasks, implying $t_a = 1/n$, $h_a = h/n$ and $y_a = A n^{-(\alpha+\beta)} h^\beta$, for all a in S . Per capita gross output, y , is found by integrating, y_a over S :

$$(A2) \quad y = \int_{a \in S} y_a da = n y_a = A n^{-(\alpha+\beta-1)} h^\beta.$$

Equation (A2) will exhibit increasing returns to specialization provided the exponent on n is negative, that is given that task-production exhibits increasing returns to scale: $\alpha+\beta > 1$. Assuming this condition is met, equation (A2) can be used to generate the following derivatives, which form the basis of assumptions regarding pre capita gross output in equation (1):

$$(A3) \quad \begin{aligned} dy/dh &> 0, \quad d^2y/dh^2 < 0, \\ dy/dn &< 0, \quad d^2y/dn^2 > 0, \\ d^2y/dndh &< 0. \end{aligned}$$

Appendix B

This appendix is divided into three sections which derive equations (17), (22) and (28), respectively.

Appendix B1

Define total per capita transaction costs as the sum of per capita internal and external transaction costs given optimal firm size. As the BB curve is monotonically decreasing and unaffected by capital accumulation, we can define optimal firm size as a function of n : $L = L(n)$, such that $[L(n), n]$ satisfies (14). It follows that $L'(n) = dL/dn|_{BB}$. Using this function, we define per capita total transaction costs as a function of n as follows:

$$(B1.1) \quad t(n) = t[n, L(n)] = \frac{I[L(n)] + E[L(n)n]}{L(n)},$$

which implies

$$(B1.2) \quad t_n(n) = \frac{(L(n)[I'[L(n)] + nE'[L(n)n]] - I[L(n)] + E[L(n)n])L'(n)}{L(n)^2} + E'(L(n)n).$$

Equation (14) implies that the numerator in equation (B1.2) is zero when evaluated at the equilibrium n , so we have

$$(B1.3) \quad t_n(n^*) = E'[L(n)n] < 0,$$

where the sign of (B1.3) follows from equation (3').

Differentiating equation (B1.2) a second time with respect to n gives

$$(B1.4) \quad t_{nn}(n) = E''[L(n)n][nL'(n)+L(n)] - L(n)^{-2} f(n)[L'(n)]^2 \\ + [L(n)]^{-1} f(n)L''(n) + [L(n)]^{-1} f(n)L'(n),$$

$$\text{where } f(n) = I'[L(n)] + nE'[L(n)n] + \frac{I[L(n)] + E[L(n)n]}{L(n)}.$$

Differentiating $f(n)$ with respect to n and rearranging the terms gives

$$(B1.5) \quad f'(n) = (I''[L(n)] + n^2 E''[L(n)n])L'(n) + L(n)nE''[L(n)n] \\ + \frac{L(n)[I'[L(n)] + E'[L(n)n]] - I[L(n)] + E[L(n)n]}{L(n)^2} L'(n)$$

Noting that equation (14) implies that the second line in equation (B1.5) is zero when evaluated at $n = n^*$, we have

$$(B1.6) \quad f'(n^*) = (I''[L(n^*)] + n^{*2} E''[L(n^*)n^*])L'(n^*) + L(n^*)n^* E''[L(n^*)n^*].$$

Equations (B1.4), (B1.6) and $f(n^*) = 0$, imply

$$(B1.7) \quad t_{nn}(n^*) = E''[L(n^*)n^*][n^*L'(n^*) + L(n^*)] + [L(n^*)]^{-1} f'(n^*)L'(n^*).$$

Substituting the formulae for $L'(n^*)$ and $f'(n^*)$ from (16) and (B1.6), respectively, into equation (B1.7) and simplifying gives the equation for $t_{nn}(n^*)$ in (17):

$$(B1.8) \quad t_{nn}(n^*) = \frac{LE''(Ln)I''(L)}{I''(L) + n^2 E''(Ln)}$$

Appendix B2

Equation (22) is derived as follows. Total differentiation of equations (13) and (14) implies

$$(B2.1) \quad \begin{bmatrix} dn / dh \\ dL / dh \end{bmatrix} = [A]^{-1} \begin{bmatrix} -y_{nh}(n, h) \\ 0 \end{bmatrix},$$

where

$$[A]^{-1} = \frac{1}{[I''(L) + n^2 E''(Ln)][y_{nn}(n, h) - t_{nn}(n)]}$$

$$\bullet \begin{bmatrix} I''(L) + n^2 E''(Ln) & nE''(Ln) \\ -LnE''(Ln) & y_{nn}(n, h) - LE''(Ln) \end{bmatrix}$$

It follows that

$$(B2.2) \begin{bmatrix} dn / dh \\ dL / dh \end{bmatrix} = \frac{1}{[I''(L) + n^2 E''(Ln)][t_{nn}(n) - y_{nn}(n, h)]}$$

$$\bullet \begin{bmatrix} [I''(L) + n^2 E''(Ln)]y_{nh}(n, h) \\ -LnE''(Ln)y_{nh}(n, h) \end{bmatrix},$$

from which we get the first two equations in (22).

To derive the third equation, note that the elasticity of z with respect to n along the BB curve is given by

$$(B2.3) (dz/dn|_{BB})n/z = (dL/dn|_{BB})(n/L) + 1 = \eta_B + 1 \in (0, 1).$$

Since the BB curve is unaffected by changes in h , we have

$$(B2.4) dz^*/dh = [z/n](dz/dn|_{BB})dn^*/dh,$$

which gives us the third equation in (22).

Appendix B3

This section of Appendix B derives equation (28), which gives the condition under which the marginal product of capital is increasing in the capital labor-ratio. Since it is assumed that workers own capital and make investment decisions, the appropriate measure of the return to capital is first derivative of the real wage with respect to the capital-labor ratio.

Noting that there is a 1-to-1 relationship between h and n^* , equations (11), (20), (21) and (B1.1) imply that the real wage as a function of the capital-labor ratio is given by

$$(B3.1) \quad w = w(h) = y[n^*(h), h] - t[n^*(h)].$$

Differentiating w with respect to h in equation (B3.1) gives the marginal product of capital as a function of h :

$$(B3.2) \quad dw/dh = y_h(n^*, h) + [y_n(n^*, h) - t_n(n^*)]dn^*/dh > 0,$$

implying that the wage is increasing in the capital-labor ratio, since the first term is positive and the second is zero if (10) holds and positive otherwise.

Differentiating a second time with respect to h gives

$$(B3.3) \quad \begin{aligned} d^2w(h)/dh^2 = & y_{hh}(n^*(h), h) + y_{hn}(n^*(h), h)[dn^*(h)/dh] \\ & + [y_n(n^*(h), h) - t_n(n^*)][d^2n^*(h)/dh^2] + y_{nh}(n^*(h), h)[dn^*(h)/dh] \\ & + [y_n(n^*(h), h) - t_n(n^*)][dn^*(h)/dh]^2 \\ & + [y_{nn}(n^*(h), h) - t_{nn}(n^*)][dn^*(h)/dh]^2, \end{aligned}$$

where the third line makes use of the fact that $d^2n^*(h)/dh^2 = dn^*(h)/dh$. Recalling that $t_n(n^*) = E'(n)$, equation (13) implies $y_n(n^*, h) - t_n(n^*) = 0$. Therefore, we have

$$(B3.4) \quad \begin{aligned} d^2w(h)/dh^2 = & y_{hh}(n^*(h), h) + 2y_{nh}(n^*(h), h)[dn^*(h)/dh] \\ & + [y_{nn}(n^*(h), h) - t_{nn}(n^*)][dn^*(h)/dh]^2. \end{aligned}$$

Substituting the formula for dn^*/dh from equation (B2.2) into equation (B3.4), we get

$$(B3.5) \quad d^2w(h)/dh^2 = y_{hh}[n^*(h), h] - y_{nh}[n^*(h), h]^2 / \{y_{nn}[n^*(h), h] - t_{nn}[n^*(h)]\},$$

which implies inequality (27).

References

- Baumgardner, James R. "The Division of Labor, Local Markets, and Worker Organization," *Journal of Political Economy*, 1988, vol. 96, no. 3.
- Becker, Gary S., and Kevin M. Murphy. "The Division of Labor, Coordination Costs, and Knowledge," *Quarterly Journal of Economics*, Nov. 1992, 57(4), pp. 1137-1160.
- Becker, Gary S., Kevin M. Murphy and Robert Tamura, "Human Capital, Fertility, and Economic Growth," *Journal of Political Economy*, Oct. 1990 (part 2), 98(5), pp. 12-37.
- Cheung, Steven, "The Contractual Nature of the Firm," *Journal of Law and Economics*, April 1983, pp. 1-21.
- Coase, Ronald H. "The Nature of the Firm." In *The Nature of the Firm*, Edited by Oliver E. Williamson and Sidney G. Winter. New York: Oxford Univ. Press, 1991.
- _____. "The Nature of the Firm: Origins." In *The Nature of the Firm*, Edited by Oliver E. Williamson and Sidney G. Winter. New York: Oxford Univ. Press, 1991.
- Grossman, Gene M. and Helpman, Elhanan. *Innovation and Growth in the Global Economy*. Cambridge, Mass.: MIT Press, 1991.
- Jones, Larry E., and Manuelli, Rodolfo, "A Convex Model of Equilibrium Growth: Theory and Policy Implications," *Journal of Political Economy*, 1990, 98(5), pp. 1008-1038.
- King, Robert G., and Rebelo, Sergio, "Public Policy and Economic Growth: Developing Neoclassical Implications," *Journal of Political Economy*, 1990, 98(5), pp. S126-S150.
- Krugman, Paul, "Complex Landscapes in Economic Geography." *American Economic Review*, May 1994, pp. 412-416.
- _____. "Increasing Returns and Economic Geography." *Journal of Political Economy*, 99(3), pp. 483-99.
- _____. "On the Number and Location of Cities." *European Economic Review*, March 1993, 37(1), pp. 293-98
- Lucas, Robert E., Jr. "On the Mechanics of Economic Development," *Journal of Monetary Economics*, July 1988, 22(1), pp. 3-42.
- Maddison, Angus, "Explaining the Economic Performance of Nations," in *Convergence of Productivity, Cross-National Studies and Historical Evidence*. Ed. William J. Baumol, Richard R. Nelson, and Edward N. Wolff. New York: Oxford Univ. Press, 1994.
- Myint, Hla. "Organizational Dualism and Economic Development," *Asian Economic Review*, 1985, 3(1), pp. 25-42.
- Rebelo, Sergio, "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy*, 1991, 99(3), pp. 500-521.
- Romer, Paul M. "Increasing Returns and Long-run Growth," *Journal of Political Economy*, Oct. 1986,

- 94(5), pp. 1002-37.
- _____. "Endogenous Technological Change," *Journal of Political Economy*, Oct. 1990, 98(5), pp. S71-102.
- Rosen, Sherwin. "Specialization and Human Capital," *Journal of Labor Economics*, 1983, 1(1), 43-49.
- Smith, Adam. *An Inquiry into the Nature and Causes of the Wealth of Nations*. 1776. Reprint. Edited by Edwin Cannan. Chicago: Univ. of Chicago Press, 1976.
- Stigler, George J. "The Division of Labor is Limited by the Extent of the Market," *Journal of Political Economy*, June 1951, 59(3), pp. 185-193.
- Williamson, Oliver E. *Markets and Hierarchies: Analysis and Antitrust Implications*. New York: The Free Press, 1975.
- _____. "Transaction-Cost Economics: the Governance of Contractual Relations" *Journal of Law and Economics*, 1979, 22, pp. 233-261.
- Yang, Xiaokai and Jeff Borland. "A Microeconomic Mechanism for Economic Growth," *Journal of Political Economy*, 1991, 99(3), pp. 460-482.
- _____. "Specialization, Product Development, Evolution of the Institution of the Firm, and Economic Growth," *Journal of Evolutionary Economics*, 1995, 5, pp. 19-42.
- Young, Allyn A. "Increasing Returns and Economic Progress," *Economic Journal*, Dec. 1928, 38, pp. 527-42.