The Division of Labor, Coordination Costs and the Growth of Government

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Abstract

This paper develops a dynamic, general equilibrium model of specialization-driven growth in which the private cost of coordination among specialists is a function of public expenditures on physical and institutional infrastructure. Growth is characterized by endogenous increases in labor specialization, capital per worker, market size, private coordination costs and government's share of total spending. By considering the role of government in facilitating an advanced division of labor, the model provides an economic explanation for the secular rise of government's share of output.

JEL Classifications- H100, O300, O410

Section 1: Introduction

While it is well known that government's share of output has risen in industrialized countries over the past century, this stylized fact has not found its way into the formal literature on growth. In part, this appears to be due to the ready availability of arguments that attribute the rise in the government's share of output to social and political causes. In *Crisis and Leviathan* (1987), for example, Higgs explains the rise of government in the US in terms of a ratchet effect caused by the interaction of periodic crises and subsequent institutional rigidity; Hughes (1993) suggests the impact of rising incomes on the taste for redistributional policies. In contrast, this paper examines the possibility that at least part of the rise in the share of government spending can be explained in terms of the purely economic role of the government in facilitating coordination in an economy with an advanced division of labor.

Yang and Boreland (1991) and Becker and Murphy (1992) construct dynamic, general equilibrium models in which growth is driven by the endogenous evolution of the division of labor.[1] At the microanalytic heart of these models are agents faced with a trade-off between increasing returns to specialization and costs of coordinating the complex set of economic interactions arising from an advanced division of labor.

This paper extends the literature on specialization-driven growth by developing a model in which an agent's private coordination costs depend on the provision of public

[1]Labor specialization has also been investigated in a variety of static contexts. Rosen (1983), Barzel and Yu (1984) and Edwards and Starr (1987) examine the incentive for labor specialization. Baumgardner (1988), Kim (1989) and Yang (1990) consider the equilibrium degree of labor specialization in static, general equilibrium models.

services. The model is used to examine the relationships between economic growth, the division of labor and the role of government in the economy, including the possibility of a virtuous cycle of growth driven by the evolution of the division of labor, capital accumulation and increases in the provision of public services.

A reliance on steady-state analysis in the formal literature on government spending and economic growth has also contributed to lack of attention to secular increases in government's share of output.[2] As the steady state is defined by the existence of constant ratios in quantity variables over time, dynamic modeling exercises, such as Uzawa (1965), Shell (1966) and Barro (1990), necessarily find that in the long run government expenditure is a constant share of output. Such as conclusion is, in fact, unavoidable: if in the long run government spending grows faster than output, it will eventually exceed total expenditure. Given the experience of the US and other industrial countries, this suggests that the "long run" of theoretical growth models must be very long and that to explain the rise of government it is necessary to consider transitional dynamics.[3]

Empirical support for the thesis that secular increases in government spending have been driven by attempts to decrease coordination costs associated with advances in the division of labor comes from Wallis and North's (1986) analysis of the composition of

[2] Though not explicitly included, a positive role for government may be inferred from the work of theorists such as Arrow (1962), Lucas (1988) and Romer (1986, 1990), who assume the existence of positive spillovers to certain investment activities and thus the divergence of private and social returns.

[3] Though not beyond critique, recent empirical work based on the neoclassical growth model supports this conclusion, regularly finding that countries and regions within a country converge toward their long run growth paths at a rate of about 2 percent per year. See Sala-i-Martin (1996) for a compilation of this evidence.

government expenditure in the US economy. Wallis and North decompose government spending into three categories in order to measure the size of the public transaction sector defined as public spending on transaction-cost reducing services. The first category consists of those expenditures directly related to the enforcement of property rights, that is expenditures on military, police and general government, and rose from 2.8 to 11.3 percent of GNP over the period from 1902 to 1970.[4]

The second category is spending on social overhead capital, which includes education, transportation and urban services. These expenditures, they argue, are indirectly linked to transaction costs. Education, for example, aids in the "socialization process regarding the legitimization of contracts," and urban services reduce the costs of living in urban areas and thus serve to facilitate the division of labor. If this second category of public expenditures is included, the public transaction sector's share of GNP increased from 5.6 to 21.6 percent over the same period.

The growth of these two categories of government spending suggests that explanations of the rising share of government spending that rely entirely on an increase taste for transfer payments are incomplete. In addition, increases in government transfer payments, Wallis and North's third category of public spending, may also be regarded as causally linked to increases in the division of labor if, as argued below, specialization results in increased political pressure for redistributional policies by special interest groups. From 1902 to 1970 government transfer payments rose from 1.6 to12.1 percent

[4]See Wallis and North, Table 3.8, p.116. As Wallis and North point out there is a certain degree of ambiguity as to whether certain government expenditures should be included in the transaction sector. With the rise of the military-industrial complex, for example, some portion of military expenditure might best be categorized as transfer payments.

of GNP.

Section 2: The Division of Labor and the Economic Role of Government

In relation to the literature on specialization-driven growth, the primary theoretical innovation in this model is the assumption that increases in the complexity of economic organization increase the return to spending on public goods that reduce the private costs of coordination. This section presents several arguments regarding the relationship between the division of labor, private coordination costs and government spending, which are intended to motivate the relationships formalized in the coordination cost equation of the model developed in the next section. Broadly speaking, these arguments may be characterized as belonging to the theories of efficient government -- statements regarding the optimal provision of public goods -- and of inefficient government – statements suggesting a relationship between advances in the division of labor and deviations from optimal public policy.

The first thesis regarding the relationship between specialization and optimal government spending is that increases in the division of labor result in increases in the return to government services that facilitate the coordination of economic activity. In particular, private coordination costs may depend negatively on the provision of public goods. For example, transportation and contract enforcement costs will be influenced by the provision of transportation infrastructure and public institutions which define and enforce property rights.[5]

[5]Increases in the division of labor will also provide an incentive for increases in the provision of private

Second, increases in the division of labor result in the substitution of formal for informal rules governing exchange and, consequently, increase the complexity and cost of administration of the legal system. According to North, "The move...from unwritten traditions and customs to written laws has been unidirectional as we have moved from less to more complex societies and is clearly related to the increasing specialization and division of labor associated with more complex societies" (1990, p. 46).

North finds the theoretical underpinnings for this claim in game-theoretic investigations of the conditions under which cooperation among individuals arises without third party intervention. In particular, a group's propensity for spontaneous cooperation is found to depend negatively on group size (Hardin 1982), positively on the density of social interactions (Taylor 1987), and negatively on the cost of obtaining information on other players' preferences (Schofield, 1985). Increases in the division of labor thus reduce an economy's ability to rely on non-formal coordination mechanisms by 1) increasing the number of individuals in each group (market or firm), 2) depersonalizing exchange by decreasing the complexity of the social network with in which exchange takes place, and 3) increasing diversity among agents, which reduces each agent's knowledge of other players' preferences.[6]

Third, increases in the division of labor lead to increases in the number and intensity of external effects and, thus, increase the scope for positive government intervention. This thesis is based on two propositions. The first is that the intensity of

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services which reduce coordination costs. See Dighe et al. (1990) for evidence regarding this effect.

[6]The existence of professional codes of behavior and the segregation of cities into "neighborhoods" based on class, race, ethnicity, religion and the like may be viewed in part as an attempt to reduce information costs between agents.

interpersonal spillover effects increases as the space between agents decreases.[7] Specific examples include traffic congestion and air and noise pollution. In general, however, the statement requires only that an agent's actions have effects that extend beyond her in space but with decreasing intensity.

The second proposition is that certain transaction costs, notably transportation and communication costs, may reasonably be thought of as decreasing in the space between transacting agents. As a result, increases in the gains to specialization will tend to increase the incentive an agent has to reduce these costs by moving closer to others with whom she has economic interactions, thereby increasing external diseconomies of propinquity.[8]

While explicit treatment of political processes is beyond the scope of the formal

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[7]. As McCloskey (1998) points out, in reference to Coase's (1960) example involving a railroad and a farmer, this conception of external effects is somewhat different from the Pigovian one in that it is not necessarily possible to determine which party is generating the externality, "the railway which makes the sparks, or the farmers who plant imprudently close to the line." It is rather from their proximity that the externality arises.

[8] While the formal model presented below does not include a spatial variable as one of the arguments of the coordination cost function, in choosing how far from others to live, optimizing agents would balance the marginal gains to specialization and the reduction of marginal transportation and communication costs against the marginal disutility of interpersonal externalities and the increase in taxes to offset them. It follows that an increase in the gains to specialization would result in a decrease in the equilibrium distance between agents, an increase in the strength of interpersonal spillover effects, and greater per capita government spending due to an increase in the marginal gains of government intervention relative to the marginal costs. Note that the existence of interpersonal spillover effects is consistent with optimal specialization decisions: agents are likely to include the psychic costs of urban living in the decision to move to a city.

analysis presented below, there are two arguments for believing that political outcomes become increasingly inefficient, from a social point of view, as the division of labor increases. First, it should be noted that as an economy develops the division of labor cannot be expected to proceed evenly within a given country. Transaction costs are not uniform across space, for example due to differences in geography such as the presence of mountains, deserts or rivers, which affect transportation costs. In addition, the gains to specialization may differ across economic activities, as Smith (1976) suggests regarding agriculture and industry.[9]

As a result, the rate at which larger trading groups, and by association larger social units such as towns and cities, emerge will vary across geographic regions and sectors of the economy. Consequently, those changes in the structure of values associated with the emergence of larger trading groups and social units, which may be referred to collectively as modernization, will also proceed at different rates. This in turn will tend to increase the divisiveness of public life as the conflict between traditional and emerging value structures is played out in the political arena.[10]

A second factor leading to decreasing efficiency of the public sector derives from Olson's work on the role of special interest groups. According to Olson political rigidities

[9]For a model of specialization-driven urbanization in which transaction costs are higher in agriculture, see Yang (1990).

[10]A particular example of this is the on-going public debate over the set of ideas and values associated with the term "feminism." As Yang and Boreland (1991) illustrate, increases in the gains to specialization decrease the incentive for household production, and may thus be seen as one of the factors contributing to increased female participation in the labor force. The association of pro- and anti-feminist stances with modern and traditional values is also clearly marked in the rhetoric of the debate.

in advanced societies are due to the accumulation of distributional coalitions, which tends to increase "the complexity of regulation, the role of government, and the complexity of understandings," i.e. formal and informal contracts (1982, p. 74). First introduced in his analysis of collective action (Olson, 1971), distributional coalitions are groups of individuals who share a common economic interest and act to further that interest through influencing public decision making to redistribute income in their favor.

Due to the collective nature of the benefits derived by the members of a distributional coalition, collective action faces free-rider problems similar to those associated with public goods. An important implication is that distributional coalitions are easier to form when the number of potential beneficiaries is small. Since redistributional policies tend to distort relative prices and thereby reduce national income, the range of redistributional policies a redistributional group will advocate depends (inversely) on how "encompassing" it is, as measured by its share of national income. For example, a group that receives half of the national income will not pursue policies that reduce national income by more than two dollars for every dollar of redistribution; a group that receives one third of national income would favor policies that reduce national income by as much as three dollars per dollar redistributed.

Olson attributes the gradual accumulation of distributional coalitions over time to the relatively high costs and uncertainty involved in coalition formation. When viewed through the lens of increasing specialization, however, a more deterministic theory of the formation and impact of coalitions may be proposed. As labor specialization proceeds professions become more numerous and more narrowly defined. Given that members of a given profession have common economic interests, this will tend to increase both the

number of potential redistributional coalitions and reduce each profession's share of national income, making it less encompassing and thus willing to pursue policies more damaging to the economy at large. If the rate of increase of labor specialization outpaces the growth of the workforce, increases in the division of labor will also tend to decrease the number of individuals in a given profession, which reduces the costs of coalition formation and facilitates coordination within the coalition once formed.[11]

Seen in this light, Olson's theory of distributional coalitions is an extension of the more general proposition that the division of labor increases the cost of coordination within an economy to the political marketplace. In addition, it necessarily places the analysis within the world of the second best. With suboptimal decisions regarding infrastructure investments and legal arrangements in the public sector of the economy, the costs of coordinating the division of labor will be higher than they would under optimal provision of public services. As it is also likely that agents' specialization decisions do not take into account their impact on the efficiency of the political process, over-specialization will tend to occur.

As the formal analysis presented below does not consider the possibility of non-optimal public decision making, these last two theses are not captured by the model. Symmetry across agents and activities implies that specialization proceeds evenly over all agents. The model, therefore, cannot be used to examine the implications of conflict

[11]There may be limits to the positive effects of reduced coalition size on coalition formation and effectiveness. In particular, if there are fixed costs to formation or lobbying, these costs must be spread among coalition members, and the benefits of membership must exceed these costs in order for the coalition to continue to exist. See Krauts (1998) on the fragmentation and decline of professional associations in the US and Europe.

between emerging and traditional value systems. A second implication of symmetry is that coalition formation takes place in all professions at a single point in time and affects the provision of public services in a symmetric manner, reducing the efficiency of public inputs but not distorting the relative prices of private goods.

Section 3: The Static Model

3.1 Basic Equations

There are N *ex ante* identical individuals, each with an exogenously given stock of capital, h, and a continuum of productive tasks arranged along the unit interval. There is a one-to-one relationship between tasks and intermediate goods, and the (measure of the) range of tasks undertaken by an individual is n. Labor specialization is denoted by the variable s, which is assumed to be inversely related to n, $s = 1/n$. That is, a worker is more specialized if she concentrates her productive efforts on an narrower range of tasks.

Following Rosen (1983) and Young (1928), increasing returns to specialization are assumed to arise from the use of specialized capital in the production of intermediate goods. By concentrating her investment resources and work effort on a limited range of intermediate goods, a worker increases the utilization rate of specialized capital and, thus, the return to capital and per capita output.

Each worker allocates her resources evenly among the tasks she undertakes and produces the same quantity of each intermediate good. Output per capita, defined as the sum of intermediate good outputs, is increasing in the degree of labor specialization and

capital per worker. We have

$$(1) \qquad y = y(s, h),$$

where $y(s, 0) = 0$,

$y_s(s, h) > 0$,

$y_h(s, h) > 0$,

$y_{ss}(s, h) < 0$,

$y_{hh}(s, h) < 0$,

$y_{hs}(s, h) > 0$ and

$0 \leq s \leq 1$.

The derivatives of equation (1) imply that per capita output is increasing and subject to diminishing returns in both labor specialization and the capital-labor ratio, and that the marginal product of capital is increasing in labor specialization. These assumptions are consistent with parametric equations for per capita output derived by Rosen (1983) and Becker and Murphy (1992) under the assumption that intermediate goods are produced using specialized capital goods.[12]

Intermediate goods are assumed to be complements in the production of a composite good that may be used either for consumption or investment. One unit of the composite good is produced in Leontief fashion by combining one unit of each of the intermediate goods. A worker may produce the full range of intermediate goods and

[12] See also the derivation of per capita output in Appendix A1, which assumes intermediate goods are produced using Cobb-Douglas technology with diminishing marginal returns to task-specific capital.

combine them to produce the composite good.[13] Alternately, she may specialize in the production of a subset of intermediate goods, in which case she must coordinate her production with other specialists in order to produce the final good.

The presence of positive gains to specialization implies that, in order to generate non-boundary solutions for the equilibrium degree of labor specialization, it is necessary to abandon the standard assumption that markets and firms function costlessly to coordinate production. The alternative assumption, that the coordination of economic activity is costly, implies that agents face a trade-off between the gains to specialization and the cost of coordination.

Coordination costs may arise for a variety of reasons. The emphasis in Adam Smith (1976) is on transportation costs, which he uses to explain the relative backwardness of inland Asia and Africa and a low division of labor among Scottish farmers, and on legal barriers to exchange, to which he ascribes the (reputed) stationarity of China. Becker and Murphy (1992) cite the recent work on principal-agent conflicts and free-rider problems as a source of coordination costs. Arrow (1974) and Coase (1937) argue that information costs associated with market transactions rise with increases in the number of transactors. Coase (1937) and Williamson (1975) call attention to the bureaucratic costs of coordinating production within firms, and Williamson (1979) stresses the role of imperfect contracting as an impediment to the coordination of production between firms. In each of these cases, coordination costs are increasing in the division of labor.

[13] The cost of combining intermediate goods is assumed to be zero. Equivalently, it is assumed that agents costlessly coordinate their own production so that $c(1, g) = 0$ in equation (2) below.

A group of agents who coordinate their production will be called a team. The number of team members, m, is thus associated with Smith's notion of the extent of the market, and increases in team membership are taken to be indicative of the emergence of larger social groupings such as towns and cities. Positive coordination costs imply that teams will consist of agents who produce non-overlapping sets of intermediate goods. It follows that team membership is given by $m = 1/n = s$. Integer problems regarding the number of agents in a team are ignored.

As argued in the previous section, private coordination costs will be decreasing in the provision of public goods. It remains to be determined what measure of government services is used in determining per capita coordination costs. As Barro (1990) points out, the correct measure of the government services available to a given agent will be per capita rather than total government expenditure if public goods are rivalrous. The prevalence of congestible public goods (highways, courts, and the like) suggests that per capita government expenditures, which is used here, may be the better measure.[14]

The function governing per capita coordination costs is given by

$$
\begin{aligned}
(2) \qquad & c = c(s, g), \\
& c_s(s, g) > 0, \\
& c_{ss}(s, g) > 0, \\
& c_g(s, g) < 0, \\
& c_{gg}(s, g) > 0 \text{ and}
\end{aligned}
$$

[14] With a fixed population, use of total government expenditure would influence the results of the model only through the introduction of a constant.

$$c_{sg}(s, g) < 0,$$

where g is per capita government spending. The derivatives in (2) imply that coordination costs are increasingly increasing in labor specialization, that successive increases in government spending reduce coordination costs but with diminishing effect, and that an increase in labor specialization increases the return to the provision of public goods.

Finally, it is assumed that the government maintains a balanced budget at every point in time, so that the government budget constraint is

$$(3) \qquad g = t,$$

where t is per capita lump sum tax revenues.

Workers are assumed to maximize net per capita income, z, which is per capita income less taxes and coordination costs. In determining equilibrium in the static model, the capital-labor ratio is taken as given, and both s and g are assumed to be chosen optimally.

Employing (3), the representative agent's optimization problem is, thus, given by

$$(4) \qquad \max_{s,\,g} \quad z(s, g, h) = y(s, h) - g - c(s, g)$$

$$\text{subject to} \quad 1 \leq s \leq N,$$

$$g \geq 0.$$

The Kuhn-Tucker conditions for maximization with respect to s and g are,

respectively,

(5a) $s \geq 1$, $y_s - c_s \leq 0$ and $(s-1)(y_s - c_s) = 0$

or

(5b) $s \leq N$, $y_s - c_s \geq 0$ and $(s-N)(y_s - c_s) = 0$,

and

(6) $g \geq 0$, $-1 - c_g \leq 0$ and $g(-1 - c_g) = 0$.

Let AA and BB, shown in Figure 1, denote the schedules defined by (5) and (6) in the s-g plane. For values of g for which the lower (upper) constraint on s is binding, the AA schedule consists of a vertical line at s = 1 (s = N). For intermediate values of g, agents choose s by equating the marginal gains to specialization, $y_s(s, h)$, with marginal cost of coordinating the division of labor. Above AA, marginal gain to specialization exceed marginal costs, and below AA marginal costs exceed marginal gains.

Differentiating $\delta z/\delta s = 0$ with respect to s and g indicates that for $1 \leq s \leq N$ the slope of the AA schedule is given by

(7) $dg/ds|_A = (y_{ss} - c_{ss})/c_{sg} > 0$,

Figure 1a: Stable Equilibrium at (s*, g*)

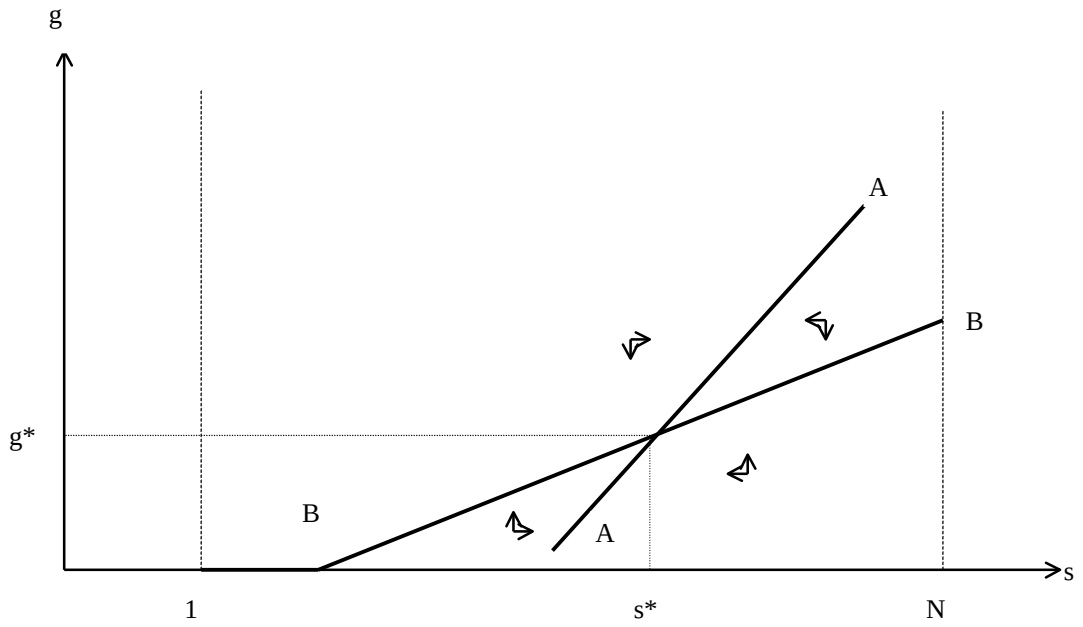
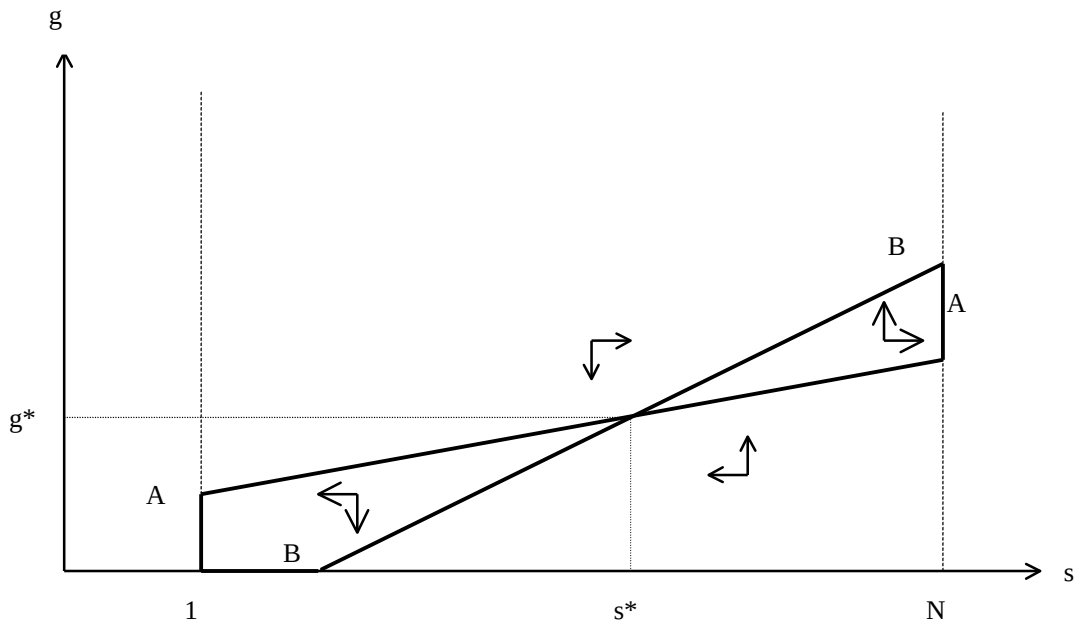


Figure 1b: Unstable Equilibrium at (s*, g*)



where the sign follows from (1) and (2). The logic behind (7) is that, starting in equilibrium, an increase in government spending decreases the marginal costs of specialization, so that at the initial value of s the marginal gains now exceed marginal costs. An increase in s is therefore necessary to re-equate the costs and gains to specialization.

Equation (6) implies that optimal positive government spending occurs when an additional dollar of tax revenue and government spending reduces coordination costs by one dollar. Since small teams may coordinate their production without resorting to third party intervention, it is assumed that the benefits of government spending in reducing coordination costs are less than one until a threshold level of specialization, s_0, is reached, and thus that optimal government spending is zero for $s < s_0$. Beyond this threshold value of s, the slope of the BB schedule is found by differentiating $\delta z/\delta g = 0$ with respect to g and s, implying

$$(8) \qquad dg/ds|_B = \begin{cases} 0 & , for\ s \le s_0 \\ -c_{sg}/c_{gg} > 0, & for\ s > s_0 \end{cases}$$

where the sign follows from (2). An increase in specialization increases the return to the provision of public goods in reducing coordination costs, implying that the benefits of additional taxation and government spending exceed the costs. Since the BB schedule is monotonic and invariant in h, we can define the equilibrium value of g as a function of s, $g^* = g^*(s)$, such that $[s, g^*(s)] \in$ BB.

3.2 Existence and Stability of Equilibria

If the AA and BB schedules intersect, indicated in Figure 1 by (s*, g*), the intersection will be an equilibrium provided the second order conditions of the agent's maximization problem (4) are met.[15] These are

(9a) $\delta^2 z/\delta s^2 = y_{ss} - c_{ss} < 0,$

(9b) $\delta^2 z/\delta g^2 = -c_{gg} < 0,$

and

(9c) $(\delta^2 z/\delta s^2)(\delta^2 z/\delta g^2) - (\delta^2 z/\delta s \delta g)^2 = -c_{gg}(y_{ss} - c_{ss}) - c_{gs}^2 > 0.$

(9a) and (9b) hold from our assumptions regarding the derivatives of (1) and (2).

We know from (7) that an increase in government spending increases the optimal degree of specialization and from (8) that an increase in specialization increases optimal government spending. The condition in (9c) implies that the interaction between these two effects is bounded. If (9c) is not met, (s*, g*) is a saddle point for z(s, g, h). In this case, optimal specialization occurs at one of the endpoints implying that equilibrium values of (s, g) are then given by (1, g*(1)) or (N, g*(N)). Since

$$-c_{gg}(y_{ss} - c_{ss}) - c_{gs}^2 = -c_{gg}c_{sg}(dg/ds|_A - dg/ds|_B) \text{ and } -c_{gg}c_{sg} > 0,$$

(9c) is satisfied provided AA is steeper than BB at (s*, g*). Unless otherwise specified, it

[15]There is, of course, nothing in the analysis or in our assumptions that precludes the existence of multiple equilibria, which will occur if the AA and BB schedules intersect more than once. Statements about the curvature of AA and BB, however, depend upon third and higher order derivatives of the objective function about which it is difficult to make reliable assumptions.

is assumed hereafter that (9c) holds.

At any equilibrium, the economy may be categorized according to its position on the BB curve as determined by the value of h and the functions y(..) and c(..). It is useful to define h_0, h_1 and h_2 as follows

(10) h_0: $y_s(1, h_0) - c_s(1, 0) = 0$

h_1: $y_s(s_0, h_1) - c_s(s_0, 0) = 0$

h_2: $y_s(N, h_2) - c_s(N, g^*(N)) = 0$

It follows that for $h \leq h_0$ the AA curve lies entirely above BB for $s > 1$, corresponding to an autarchic equilibrium at $(s^*, g^*) = (1, 0)$. In autarchy, each agent produces the full range of intermediate goods and is economically independent. In addition, in the absence of specialization coordination costs are zero and there is no government.

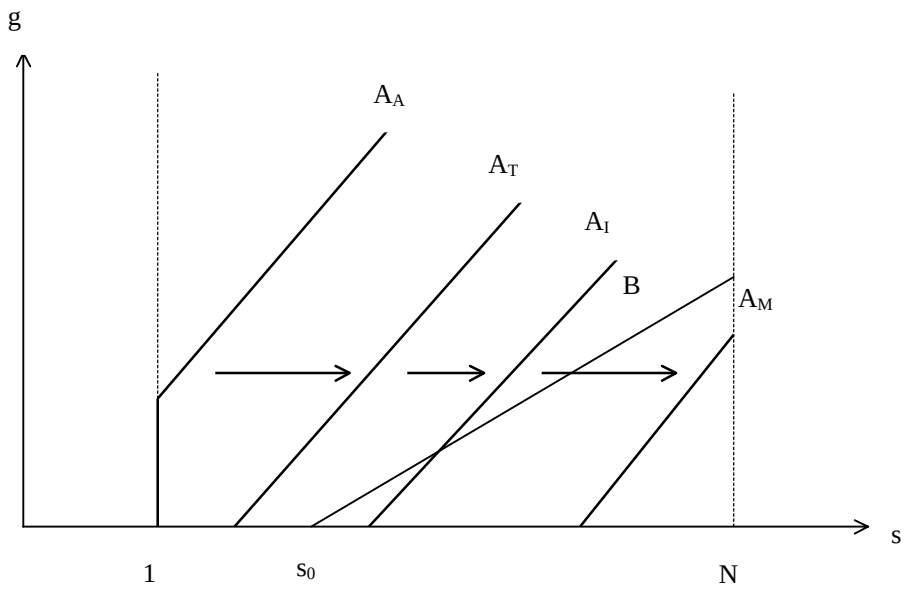
For $h \in [h_0, h_1]$, the equilibrium lies on the flat portion of BB, corresponding to a traditional economy. Traditional economies are characterized by the existence of many small teams, low-levels of the division of labor and the absence of government, social groups being sufficiently small for coordination of rely on informal enforcement mechanisms.

For $h \in (h_1, h_2)$, the equilibrium lies on the upward sloping portion of the BB curve. In this case, the economy consists of N/s^* teams of $m = s^*$ individuals, and positive government spending. Along this portion of the BB curve, labor specialization is positively related to team size and per capita government spending and negatively related to the number of teams. Greater specialization results in greater economic integration

interdependence among agents and increases in the role of government in reducing private coordination costs.[16] These equilibria are interpreted as corresponding to an industrializing economy.

Finally, for $h > h_2$, the AA curve lies entirely below the BB curve. Specialization is constrained by population size rather than the cost of coordinating the division of labor, implying an equilibrium at $(s^*, g^*) = (N, g^*(N))$. In this case the economy is fully integrated and consists of a single team, corresponding to a mature economy.

Figure 2: Position of the AA Curve for Various Values of h



[16] Economic integration predicted by the model may also play a role in the centralization of government. Hughes (1993) reports that since 1929 in the US total government spending has grown 4 times as fast as GNP while spending by the federal government has grown 7 times as fast, rising from 3 percent of GNP in 1929 to between 20 and 25 percent of GNP in 1987.

3.3 Comparative Statics

As illustrated in Figure 2, an exogenous increase in the capital stock shifts the AA curve down and to the right. Comparative statics are, then,

$$\text{(11a)}\quad ds^*/dh = \begin{cases} 0 & , for\ h \in [0, h_0) \\ \dfrac{-y_{sh}}{y_{ss} - c_{ss}} > 0 & , for\ h \in [h_0, h_1) \\ \dfrac{-c_{gg}\, y_{sh}}{\Delta} > 0 & , for\ h \in [h_1, h_2) \\ 0 & , for\ h \geq h_2 \end{cases}$$

and

$$\text{(11b)}\quad dg^*/dh = \begin{cases} \dfrac{c_{sg}\, y_{sh}}{\Delta} > 0 & , for\ h \in [h_1, h_2) \\ 0 & , otherwise \end{cases}$$

where $\Delta = (y_{ss} - c_{ss})c_{gg} + c_{sg}^2 < 0$.[17]

Starting in autarchy and allowing h to increase, specialization begins at $h = h_0$. With further increases in the capital-labor ratio, specialization and the gains to third party enforcement of contracts rise, with government formation occurring at $h = h_1$. Additional increases in h result in increases in labor specialization, the interdependence of agents and per capita government spending until $h = h_2$, after which the division of labor is

[17] See Appendix A2 for derivation.

constrained by the size of the population and increases in the capital-labor ratio affect neither g nor s.

The second line of (11a) shows the horizontal shift in the AA curve and, thus, the increase in s in the absence of government. This effect will be larger the more sensitive the gains to specialization are to the capital-labor ratio. In addition, the effect will be large when the difference in the rates of increase of marginal costs and returns to specialization is small, since this implies that a large change in s is necessary to restore the balance between the marginal costs and marginal gains to specialization.

Since government spending may be used to offset rising coordination costs, the impact of a given change in h will be larger in an industrializing than a traditional economy. This may be seen by noting that the third line of (11a) is the horizontal shift in the AA curve multiplied by

$$\left[1-\frac{dg/ds|_B}{dg/ds|_A}\right]^{-1},$$

which is greater than 1 given our assumption regarding the relative slopes of the AA and BB curves. This term captures the positive interaction between specialization and optimal government spending, an interaction which is absent in the second line of (11a) since government spending is constrained at 0. This effect will be larger the greater c_{sg}, which indicates the magnitude of the reduction in coordination costs due to an increase in government spending.

Rearranging terms in (11b), we see that for $h \in [h_1, h_2]$ the increase in government spending is given by

$$dg^*/dh = 0\left(dg/ds|_B\right)\left(ds^*/dh\right).$$

Here, both terms are increasing c_{sg}, indicating that the effect of an increase in the capital-labor ratio on government spending is larger the greater the effectiveness of government spending in reducing marginal coordination costs.

Section 4: Endogenous Growth and the Rise of Government

This section introduces an informal dynamic model to consider the evolution of the economy through time. In addition, we derive the conditions under which government spending grows as a share of output and note the relationship between the growth of government and diminishing marginal returns.

4.1 Endogenous Growth

By allowing the final composite good to be either invested or consumed and positing agents with an infinite time horizon, the static model developed in Section 3 may be extended to consider issues of economic growth. Of primary interest here are the ability of the model to generate endogenous and the role of increasing government spending in this process. This contrasts with the main line of endogenous growth theory which has argued that the special properties of either knowledge or human capital make the assumption of diminishing marginal returns to capital inappropriate.

An informal dynamic model is constructed by assuming that the rate of investment conforms to the following function:

(12) $dh/y = f[z'(h)-\theta]$,

where $f'(..) > 0$, $\theta > 0$ is the discount rate and $z(h) = z[s^*(h), g^*(h), h]$.

Equation (12) captures two properties of investment functions resulting from dynamic utility maximization exercises with an isoelastic utility function. First, investment is a positive function of the difference between the marginal product of capital and the discount rate, which is the familiar Keynes-Ramsey rule. The second property, consumption smoothing over time, implies the gradual adjustment of the capital-labor ratio to its equilibrium value.[18] It follows from equation (12) that the model generates endogenous growth if the marginal product of capital is greater than the discount rate and increasing in the capital-labor ratio.

The growth process is driven by mutually reinforcing increases in the capital-labor ratio and the division of labor. A rise in the capital-labor ratio has both direct and indirect effects on the marginal product of capital. The direct effect, due to the presence of diminishing marginal returns to capital, is negative. The indirect effect results from the impact of capital accumulation on labor specialization and that of labor specialization on the marginal product of capital.

Recall that the gains to specialization derive from the fact that specialization allows agents to concentrate their capital endowment and working time on a narrower range of tasks increasing both capital per task and the utilization rate of task-specific capital. As a result, the return to capital is increasing in the degree of specialization. In addition, as shown in the first part of this section, the equilibrium degree of specialization is rising in

[18]For use of a similar reduced-form investment equation, see Becker, Murphy and Tamura (1992).

the capital-labor ratio.[19] Thus, an increase in the capital-labor ratio increases the gains to specialization, resulting in a higher equilibrium degree of specialization and an increase in the marginal product of capital.

If the indirect effect is sufficiently strong to offset diminishing marginal returns to capital, the marginal product of capital will be increasing in the capital-labor ratio. A precise statement of this condition is given by

$$(13) \qquad y_{sh}(s^*, h)\left(\frac{ds^*}{dh}\right) > -y_{hh}(s^*, h) \ .^{20}$$

The right hand side of (13) is a measure of the strength of diminishing marginal returns to capital. The left hand side shows the indirect effect to be the product of the marginal increase in the return to capital due to an increase in the degree of labor specialization and the increase in the equilibrium degree of labor specialization due to an increase in the capital-labor ratio, as derived in equation (11a). Thus, the model exhibits aggregate increasing returns provided the interaction between capital accumulation and economies of specialization is strong relative to the effects of diminishing marginal returns to capital.

The dynamic evolution of the economy is determined by the range of values of h for which the condition in (13) is met. Clearly, (13) fails to hold for autarchic and mature

[19]Mathematically, the increase in the return to specialization due to an increase in the capital labor ratio and the increase in the marginal product of capital due to an increase in labor specialization are identical, given by $-y_{nh}(n, h)$ and $-y_{hn}(n, h)$, respectively.

[20]See Appendix B3 for derivation.

economies since ds*/dh, and thus the left hand side of (13), is zero. Consequently, the model is dominated by diminishing marginal returns to capital for $h < h_0$ and $h > h_2$.

One possibility is that (13) holds for $h \in [h_0, h_2]$. In this case, the model exhibits increasing marginal returns to capital for $h \in [h_0, h_2]$. Defining $\theta_0 = z'(h_0)$ and $\theta_2 = z'(h_2)$, the model has up to three equilibria as follows:

(14a) $\theta > \theta_2$ one stable equilibrium at h_L

(14b) $\theta \in [\theta_0, \theta_2]$ stable equilibria at h_L and h_H
one unstable equilibrium at h_U

(14c) $\theta < \theta_0$ one stable equilibrium at h_H,

where $h_L = z'(\theta)^{-1}$, for $h_L < h_0$,

$h_U = z'(\theta)^{-1}$, for $h_U \in [h_0, h_2]$,

$h_H = z'(\theta)^{-1}$, for $h_H > h_2$.

Here h_L is a stable low-level equilibrium occurring in autarchy, and h_H is a stable high-level equilibrium in a mature economy. h_U is a threshold level of the capital-labor ratio below which the economy is driven to the low-level equilibrium. Figure 3.1 shows these equilibria given $\theta \in [\theta_0, \theta_2]$.

Since an increase in government spending may be used to offset rising coordination costs, ds*/dh will tend to be greater once government formation begins at h_1. A second possibility is, thus, that the marginal product of capital decreases over $[h_0, h_1]$ with increasing returns over $h \in [h_1, h_2]$. The implications of this case differ from that discussed above only in that the necessary and sufficient condition for the existence of a low-level equilibrium is $\theta > \theta_1 = z'(h_1)$. Provided this condition is met, the low-level

Figure 3a: Case 1, Stable and Unstable Equilibria, $\theta \in (\theta_0, \theta_2)$

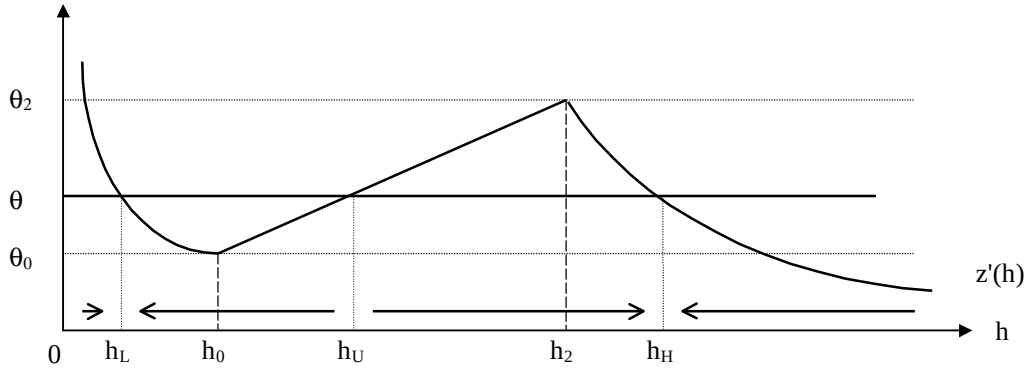


Figure 3b: Case 2 Stable and Unstable Equilibria, $\theta \in (\theta_1, \theta_2)$

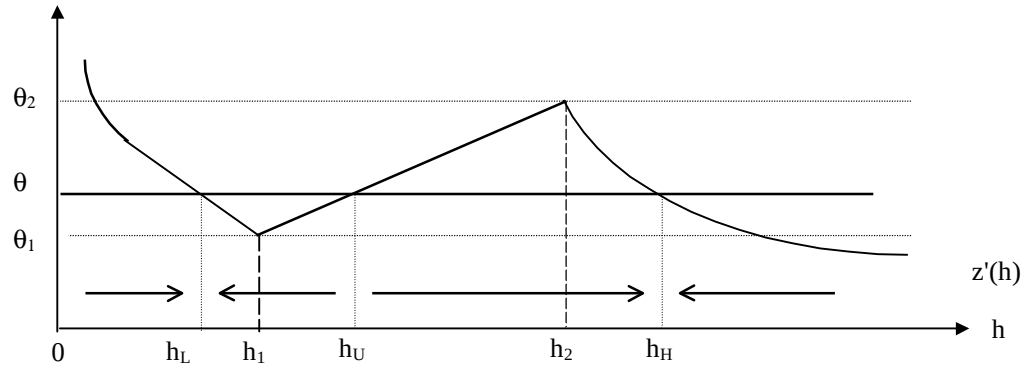
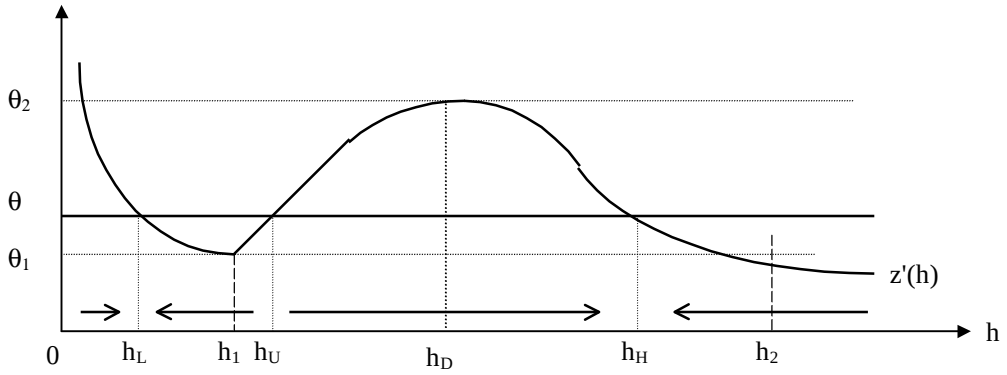


Figure 3c: Case 3 Stable and Unstable Equilibria, $\theta \in (\theta_1, \theta_2)$



equilibrium may occur at any value of $h < h_1$, as shown in figure 3.2. That is, both autarchic and traditional economies may experience low-level equilibria.

In these cases, both specialization and per capita government expenditures will increase as growing economy approaches complete specialization. Once the capital-labor ratio reaches h_2, however, specialization is limited by population size. With a fixed division of labor, coordination costs cease to increase and, as a result, agents have no incentive to increase per capita government expenditure. As additional increases in the capital-labor ratio cause per capita output to rise, government expenditure declines as a share of total output as a completely specialized economy approaches the high-level equilibrium.

A third possibility is that increases in the division of labor decrease the effectiveness of government spending in reducing coordination costs. In this case, the model may be used to illustrate a version of Olson's argument that the accumulation of distributional coalitions leads to a deceleration of economic growth and eventual economic stagnation.[21] As argued earlier, the number of potential coalitions, the ease of coalition formation and the divisiveness of coalitions in public life are likely to increase with the division of labor as the number of individuals in each profession decreases and professions become less encompassing.

Decreases in the ability of the public sector to facilitate the coordination of economic activity reduce the magnitude of the indirect effect in equation (13), eventually resulting in the onset of diminishing marginal returns to capital and undermining the

[21]This is only a "version" because it ignores significant elements of Olson's argument, most significantly the possibility of non-optimal political outcomes, non-symmetries in the impact of coalitions on public policy and other mechanisms through which coalitions affect economic outcomes, e.g. price fixing.

virtuous cycle of growth due to increases in the division of labor, capital accumulation and increases in the provision of public goods.

More specifically, assume

(15a) $\lim (s \rightarrow \infty)\ c_{sg}[s, g^*(s)] = 0$

(15b) $\dfrac{y_{sh}^2}{c_{ss} - y_{ss}} < - y_{hh}$

(15c) $h_H = z'(\theta)^{-1}$, for $h_H \in (h_U, h_2)$.

(15a) and (15b) imply that the absolute value of $c_{sg}(s, g)$ falls as s increases and that increases in government spending are necessary to generate increasing marginal returns to capital, so that diminishing marginal returns hold for $h < h_1$ and $h > h_D$, where h_D is defined implicitly by equality in equation (13). (15c) implies that specialization is limited by diminishing returns to the provision of public goods prior to complete specialization, and thus that the high-level equilibrium occurs at $h_H < h_2$ as indicated in figure 3.3.

Diminishing marginal returns may, of course, set in for reasons other than the decreasing ability of governments to reduce private coordination costs through additional public spending. For example, if the stock of knowledge is fixed it may place a limit on the accumulation of specialized human capital goods and, thus, on the gains to specialization beyond some level of the capital-labor ratio. More generally, diminishing returns may result from any factor that causes the left-hand side of (13) to fall relative to the right. The explanation of diminishing marginal returns outlined above, however, is important because it coincides with the explanation for economic sclerosis in mature

economies provided by informed observers.

While the model does not explicitly solve for the rate of growth at any point in time, it is possible to make qualitative statements regarding growth rates. In the steady states indicated by the high and low-level equilibria, the division of labor is constant and the economy stationary. Outside the steady state, the rate of per capita income growth is a positive function of the difference between the return to capital and the discount rate. This owes to two effects: a higher return to capital implies 1) a greater share of investment in output, as indicated by equation (12), and 2) a greater increase in per capita output per incremental increase in the capital-labor ratio. As a result, the model predicts that middle income countries will grow the faster than rich countries or poor countries, a prediction that is consonate, in a broad manner, with the observed pattern of growth rates across countries.[22]

4.2: On the Shares of Government Expenditure and Coordination Costs in Output

Here, we consider the conditions under which the share of government spending and of coordination costs rise in a growing economy. As the dynamics of the model are driven by capital accumulation, this is accomplished by comparing the elasticities of government expenditure, coordination costs and output with respect to the capital-labor

[22] A review of the empirical work on growth is beyond the scope of this paper. However, it should be noted that both the neoclassically motivated empirical literature on growth, for example Barro and Sala-i-Martin (1992), Mankiw, Romer and Wiel (1992) and Sala-i-Martin (1996), and the more recent work on convergence clubs, for example Quah (1996), suggest that poor countries are converging toward low-level equilibria while middle income countries catch up with rich ones.

ratio. This task is simplified by imposing two additional restrictions on the model.

First, we assume that per capita output is a Cobb-Douglas function of h and s with output elasticities $\alpha \equiv \eta(y, s)$ and $\beta \equiv \eta(y, h)$,

where $\alpha, \beta \in (0, 1)$ and $\eta(f, x) \equiv \frac{\partial f(x)}{\partial x}\frac{x}{f(x)}$. Second, it is assumed that c(s, g) is homogeneous of degree r >1 in s and g. The second assumption implies that for a given increase in s, a more than proportionate increase in government spending is necessary to reduce coordination costs to their initial level. This would tend to hold, for example, if increases in the division of labor undermine informal coordination mechanisms.

An increase in the capital-labor ratio will increase the shares of government and coordination costs in total output, respectively, provided

$$\text{(16a)} \qquad \eta(g, s)_B = -\frac{\eta(c_g, s)}{\eta(c_g, g)} > \frac{1+\eta(c_s, s)}{1-\eta(c_s, g)}$$

and

$$\text{(16b)} \qquad \eta(g, s)_B = -\frac{\eta(c_g, s)}{\eta(c_g, g)} > 1.$$

Here, (16a) implies that cross elasticities are strong relative to own elasticities.[23]

[23]The second order condition for an internal maximum given by (9c) implies a limit on the magnitude of cross effects relative to own effects. Both (9c) and (12) will be met provided

$$\frac{1+\eta(c_S, s)}{1-\eta(c_S, g)} > \alpha.$$

Intuitively, it will be satisfied provided government spending is relatively effective at reducing coordination costs. (16b) holds due to $r > 1$.[24]

Interestingly, further manipulation reveals that the model does not suggest a causal relationship between the growth of the public sector and the onset of diminishing returns. In fact, while there is nothing in the model that precludes the possibility that increases in government's share of expenditure coincide with the onset of diminishing marginal returns, the two tend to be inversely related. The conditions under which the share of government increases and the economy experiences diminishing marginal returns are, respectively,

$$(17a) \qquad \eta(g, s)_B \eta(s, h) > \alpha\eta(s, h) + \beta$$

and

$$(17b) \qquad 1 > \alpha\eta(s, h) + \beta$$

where (17b) is the counterpart to (13) restated in terms of elasticities. *Ceteris parabus*, (17a) will tend to hold when the elasticity of specialization with respect to the capital-labor ratio, $\eta(s, h)$, is large, and (17b) when it is small.

This result is due to two characteristics of the model. First, the use of lump sum, rather than marginal, taxes precludes the growth of government spending having a direct effect on the marginal product of capital. More importantly, the assumption that the level of government spending is chosen optimally implies that agents choose higher levels of government spending only if the marginal productivity of public spending is relatively high. As a result, a decrease in the productivity of public goods will tend to reduce the growth

[24] See Appendix B2 for derivations.

rates of both output and government spending.[25]

Section 5: Conclusion

The principle focus of this paper has been to develop a formal model of economic growth capable of explaining the secular growth of government in developed countries. In doing so, it departs from the familiar assumptions that markets function costlessly and that the institutional context within which market transactions occur may be taken as given and relegated to the background. Instead, the model assigns an explicit role for the government in providing the physical and institutional infrastructure necessary to coordinate economic activity involving an advanced division of labor. Thus, the economic role of government spending is placed in the foreground, with optimal public spending at each point in time determined endogenously, along with labor specialization, by the structure of the production and coordination cost functions and the capital-labor ratio.

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[25] It is, however, possible for both (17a) and (17b) to be satisfied, a sufficient condition for which is

$$-\frac{\eta(c_g,s)}{\eta(c_g,g)} > \frac{1+\eta(c_s,s)}{1-\eta(c_s,g)} > \frac{\alpha}{1-\beta}.$$

The first inequality is from (12) and implies that an increase in the capital-labor ratio increases government's share of output. The second is a sufficient condition for (17b) to be satisfied provided the first inequality holds and, as expected, will tend to hold when β is small, implying that diminishing marginal returns to capital set in quickly, and the gains to specialization, α, are small.

The dynamic model developed in Section 4 produces stable low-level equilibria in autarchic and traditional economies and stable high-level equilibria in economies with complete specialization or an advanced division of labor. As I have shown in a similar model (under review for publication), the “sideways-S” shaped function for the return to capital generated by the model is consistent with divergence between high and low income countries and convergence on the part of middle income countries.

The model also produces the somewhat curious outcome that the growing share of government expenditure is not the result of diminishing returns to public goods in defraying private coordination costs. As suggested at the end of the last section, this probably owes to the assumption of optimality in public spending decisions. By implication, if the rising share of government spending in mature economies is related to their declining rate of growth, the explanation would seem to belong to the theory of inefficient government, such as the link between specialization and the impact of distributive coalitions discussed above.

The approach used here employs much of the conceptual framework of neoinstitutionalist economics. Both are primarily concerned with issues of complexity and coordination in economic life, employ a "nano-economic" analytical framework which takes the individual economic transaction as the fundamental unit of analysis, and posit a central role for transaction costs in understanding the organization of economic activity.

An important difference is that the central focus of neoinstitutionalist attention has been structure of individual transactions, in particular whether a given transaction is best organized through the market or within a firm. The analysis here takes as its subject the organization of economy as a whole and, more importantly, the evolution of organization

over time. In addition, while the neoinstitutionalists may be interpreted as saying that "transaction costs matter," the model developed here suggests why they are important. Transaction costs matter because coordination matters, and coordination matters because of gains to the division of labor.

Appendix A1

This appendix derives a parametric function for per capita output using a Cobb-Douglas production function for intermediate goods with specific inputs. The parametric function is shown to support the assumptions made in Section 3 regarding the signs of the derivatives of equation (1).

Let the set of tasks undertaken by an individual be given by $S \subseteq [0, 1]$. The (measure of the) number of tasks undertaken is given by n:

$$n = \int_0^1 b_a da,$$

where $b_a = 1$ if $a \in S$ and 0 otherwise.

There is a one-to-one relationship between tasks and intermediate goods: performing a task produces a quantity, y_a, of the intermediate good of the same index number. Intermediate goods are produced according to a Cobb-Douglas production function with arguments t_a and h_a, which are respectively the time and capital allocated to task a:

$$(A1) \quad y_a = A t_a^{\alpha} h_a^{\beta},$$

where $\alpha, \beta \in (0, 1)$ and are uniform across tasks.

Each individual is endowed with a single unit of time and h units of capital which is allocated evenly among tasks. Defining the degree of specialization $s \equiv 1/n$ implies $t_a = s$, $h_a = sh$ and $y_a = As^{(\alpha+\beta)}h^{\beta}$, for all a in S. Per capita output, y, is found by integrating, y_a over S:

$$(A2) \quad y = \int_{a \in S} y_a \, da = y_a/s = As^{(\alpha+\beta-1)}h^{\beta}.$$

Equation (A2) implies that per capita output exhibits increasing returns to specialization provided the exponent on s is negative, that is provided task-production exhibits increasing returns to scale: $\alpha+\beta > 1$. Assuming this condition is met, equation (A2) can be used to generate the following derivatives, which form the basis of assumptions regarding pre capita output in equation (1):

$$(A3) \quad dy/dh > 0, \; d^2y/dh^2 < 0,$$
$$dy/ds > 0, \; d^2y/ds^2 < 0,$$
$$d^2y/dsdh > 0.$$

Appendix A2

This appendix derives equations (11a) and (11b). To begin with, note that equilibrium is given by $(s^*, g^*) = (1, 0)$, for $h < h_0$, so ds^*/dh and dg^*/dh are obviously zero. A similar argument shows $dg^*/dh = ds^*/dh = 0$ for $h > h_2$. In addition, h_1 is defined such that it is the greatest value of h for which $g = 0$. Consequently, $dg/dh = 0$ for $h < h_1$. With dg/dh constant, it follows that for $h \in (h_0, h_1)$, ds^*/dh may be found by differentiating $dz/ds = 0$ with respect to h and s, which yields the second line of (11a):

$$ds^*/dh = \frac{-y_{sh}}{y_{ss} - c_{ss}}.$$

Finally, the third line of (11a) and first line of (11b) are found by solving the system of equations generated by total differentiation of dz/ds = 0 and dz/dg = 0:

$$\begin{bmatrix} y_{ss} - c_{ss} & -c_{sg} \\ c_{sg} & c_{gg} \end{bmatrix} \begin{bmatrix} ds \\ dg \end{bmatrix} = \begin{bmatrix} -y_{sh}\,dh \\ 0 \end{bmatrix},$$

which yields

$$\begin{bmatrix} ds \\ dg \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -c_{gg}\,y_{sh}\,dh \\ c_{sg}\,y_{sh}\,dh \end{bmatrix}$$

where

$$\Delta = \begin{vmatrix} c_{gg} & c_{sg} \\ -c_{sg} & y_{ss} - c_{ss} \end{vmatrix} = (y_{ss} - c_{ss})c_{gg} + c_{sg}^2 < 0,$$

and the sign of Δ follows from (9c).

Appendix B1

This appendix derives equation (13). As argued in the body of the paper, for the model to generate endogenous growth the marginal net product of capital must be non-decreasing in the capital-labor ratio. An equivalent condition is that the second derivative of net output with respect to h is non-negative. Differentiating z(h) = z[s*(h), g*(h), h] twice with respect to h, we have

(B1) $z''(h) = y_{hh} + 2s'(h)y_{sh} + \{y_{ss} - c_{ss}\}s'(h) - 2c_{sg}\,g'(h)s'(h) - c_{gg}\,g'(h)^2$

Recalling from (11) that $g'(h) = -\frac{c_{sg}}{c_{gg}} s'(h)$, the last three terms of (B1) simplify to

$$\left\{ y_{ss} - c_{ss} + \frac{c_{sg}^{\ 2}}{c_{gg}} \right\} s'(h)^2 = -s'(h) y_{sh},$$

from we get $z''(h) = y_{hh} + s'(h) y_{sh}$ and the inequality in (13).

Appendix B2

This appendix derives the inequalities in (16a), (16b) and footnote 24. We start with the assumption made in the body of the paper that that per capita output is a Cobb-Douglas function of h and s with output elasticities $\alpha \equiv \eta(y, s)$ and $\beta \equiv \eta(y, h)$, where $\alpha, \beta \in (0, 1)$ and $\eta(f, x) \equiv \frac{\partial f(x)}{\partial x} \frac{x}{f(x)}$. This implies the following elasticities

(B3) $\eta(y_h, s) = \alpha$

$\eta(y_s, s) = \alpha - 1$

$\eta(y_s, h) = \beta$

$\eta(y_h, h) = \beta - 1$

From (11), it follows that the elasticities of government spending, coordination costs and output with respect to h are given, respectively, by

(B4) $(dg/dh)(g/h) = \eta(g, s)_B \eta(s, h)$

$(dc/dh)(c/h) = \{\eta(c, s) + \eta(c, g) \eta(g, s)_B\} \eta(s, h)$

$$(dy/dh)(y/h) = \alpha\eta(s, h) + \beta$$

where $$\eta(s, h) = \frac{\beta}{-h(c_s, g)[\eta(g, s)_A - \eta(s, h)_B]}$$

$$\eta(g, s)_A = \frac{\eta(y_s, s) - \eta(c_s, s)}{\eta(c_s, g)}$$

$$\eta(g, s)_B = -\frac{\eta(c_g, s)}{\eta(c_g, g)}$$

It follows that government's share of expenditure increases provided

$$\eta(g, s)_B\eta(s, h) > \alpha\eta(s, h) + \beta$$

Dividing both sides by $\eta(s, h)$ and substituting for $\eta(s, h)$ and $\eta(g, s)_A$ from the formulae above, we have

$$\eta(g, s)_B\eta(s, h) > \alpha + \eta(c_s, s) - \eta(y_s, s) - \eta(c_s, g)\eta(g, s)_B,$$

which, recalling that $\eta(y_s, s) = \alpha - 1$, implies

(B5) $$\eta(g, s)_B > \frac{1 + \eta(c_s, s)}{1 - \eta(c_s, g)}$$

which is the inequality in (16a).

As noted in the body of the paper, (B5) will hold provided cross effects are strong relative to own effects in c(s, g). From (9c), however, we know that cross effects cannot be too strong if the model is to generate non-boundary equilibria. It remains to be shown under what conditions both of these requirements may be satisfied.

As noted earlier, (9c) will be met provided the AA curve is steeper than the BB curve at (s*, g*). An equivalent requirement is that $(g, s)_A > \eta(g, s)_B$. Therefore, if (B5a) is met, (9c) will be satisfied provided

$$\frac{\eta(y_s, s) - \eta(c_s, s)}{\eta(c_s, g)} > \frac{1 + \eta(c_s, s)}{1 - \eta(c_s, g)}.$$

Substituting $\eta(y_s, s) = \alpha - 1$ this reduces to

$$\frac{1+\eta(c_s,s)}{1-\eta(c_s,g)} > \alpha,$$

which is the condition given in footnote 24.

From (B4), capital accumulation will increase the share of coordination costs in total expenditure provided

(B6a) $\eta(c, s) + \eta(c, g)\eta(g, s)_B > \alpha + \beta\eta(s, h)^{-1}$

or, equivalently,

(B6b) $\eta(g, s)_B > \frac{1+\eta(c_s,s)-\eta(c,s)}{\eta(c,g)-\eta(c_s,g)}$

The inequality in (B6b) will be satisfied provided c(s, g) is homogeneous of degree $r > 1$, which implies

(B7) $\eta(c, s) + \eta(c, g) = r$

$\eta(c_s, s) + \eta(c_s, g) = r-1$

$\eta(c_g, s) + \eta(c_g, g) = r-1$

Taken together, the first two lines of (B7) imply

$$\eta(c, s) + \eta(c, g) = 1+ \eta(c_s, s) + \eta(c_s, g)$$

or

$$\eta(c, g) - \eta(c_s, g) = 1+ \eta(c_s, s) - \eta(c, s),$$

from which it follows that the right hand side of (B6b) is equal to one. In addition, from the third line of (B7) and $r > 1$, we know

$$\eta(c_g, s) + \eta(c_g, g) > 0$$

or

$$\eta(c_g, s) > -\eta(c_g, g),$$

which implies that the right hand side of (B6b) is greater than one, since $\eta(c_g, g)$ is positive, and thus that (B6b) is satisfied.

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